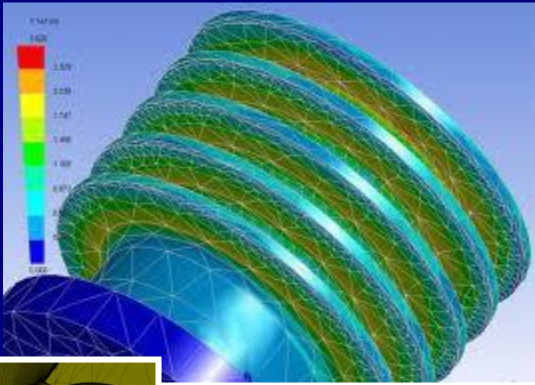
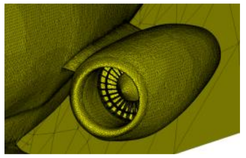
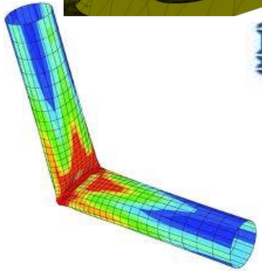
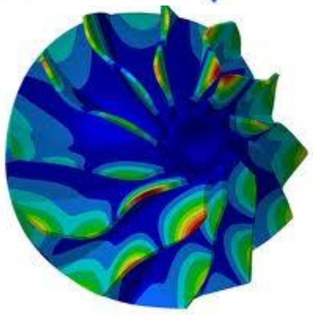
I.I.Vorovich Institute
Mathematics
Mechanics
Computer Science

Using FlexPDE finite element program









Dr. Anna A. Nasedkina
Southern Federal University

Outline of lecture 4 “Using FlexPDE finite element program”

- What is FlexPDE, what it can do and who can use it
- FlexPDE script
- Boundary conditions
- Basic steps to solve a problem in FlexPDE
- Variables and equations
- Domain description: regions, boundary paths (lines, splines, arcs)
- Finite element mesh generation
- Defining material properties
- Graphical output

What is FlexPDE?

- FlexPDE is a "scripted finite element model builder and numerical solver". This means that from a script written by the user, FlexPDE performs the operations necessary to turn a description of a partial differential equations system into a finite element model, solve the system, and present graphical output of the results.
- FlexPDE is also a "problem solving environment", because it performs the entire range of functions necessary to solve partial differential equation systems: an editor for preparing scripts, a mesh generator for building finite element meshes, a finite element solver to find solutions, and a graphics system to plot results.
- FlexPDE has no pre-wired problem domain or equation list. <u>The choice of partial differential equations is totally up to the user</u>.

What can FlexPDE do?

- FlexPDE can solve systems of *first- or second-order partial differential equations* in Cartesian or axi-symmetric two-dimensional geometry or in three-dimensional Cartesian geometry.
- The system may be steady-state or time-dependent, or alternatively FlexPDE can solve eigenvalue problems.
- Any number of simultaneous equations can be solved, subject to the limitations of the computer on which FlexPDE is run.
- The equations can be linear or nonlinear. Nonlinear systems are solved by applying a modified Newton-Raphson iteration process.
- Any number of regions of different material properties may be defined. Modeled variables are assumed to be continuous across material interfaces.

Who can use FlexPDE?

- <u>Researchers</u> in many fields can use FlexPDE to model their experiments or apparatus, make predictions or test the importance of various effects. Parameter variations or dependencies are not limited by a library of forms, but can be programmed at will.
- <u>Engineers</u> can use FlexPDE to do design optimization studies, feasibility studies and conceptual analyses. The same software can be used to model all aspects of a design; no need for a separate tool for each effect.
- <u>Educators</u> can use FlexPDE to teach physics or engineering. A single software tool can be used to examine the full range of systems of interest in a discipline. <u>Students see the actual equations</u>, and can experiment interactively with the effects of modifying terms or domains.

What does a script look like?

- TITLE – a descriptive label for the output.
- SELECT – user controls over the default behavior of FlexPDE
- VARIABLES – here the dependent variables are named.
- DEFINITIONS – useful parameters, relationships or functions are defined.
- EQUATIONS – each variable is associated with a partial differential equation.
- INITIALVALUES – starting values for nonlinear or time-dependent problems.
- BOUNDARIES – the geometry is described by walking the perimeter of the domain, stringing together line or arc segments to bound the figure.
- PLOTS – the desired graphical output is listed. Plots may be any combination of CONTOUR, SURFACE, ELEVATION or VECTOR plots.

Notation

❑**Comments** can be placed anywhere in a script.
{ Anything inside curly brackets is a comment. }
from an exclamation to the end of the line is a comment.

❑ Differentiation, such as du/dx, is denoted by the form dx(u). All active coordinate names are recognized, as are second derivatives like dxx(u) and differential operators Div, Grad and Curl.

❑ Names are NOT case sensitive. "F" is the same as "f".

Example of a script

```
TITLE 'Simple diffusion equation'
{ this problem lacks sources and boundary conditions }
VARIABLES
phi
DEFINITIONS
k=3 { conductivity }
EQUATIONS
div(-k*grad(phi)) =0
BOUNDARIES
REGION 1
START(0,0)
LINE TO (1,0)
TO (1,1)
TO (0,1)
TO CLOSE
PLOTS
CONTOUR(phi)
VECTOR(-k*grad(phi))
END
```

Boundary conditions

The primary types of boundary condition are VALUE and NATURAL

- The VALUE (or Dirichlet, essential) boundary condition specifies the value that a variable must take on at the boundary of the domain.
- The NATURAL boundary condition specifies a flux at the boundary of the domain. (The precise meaning of the NATURAL boundary condition depends on the PDE for which the boundary condition is being specified.)

```
BOUNDARIES
REGION 1
START(0,0)
VALUE(u) = 0 LINE TO (1,0) { fixed value
on bottom }
NATURAL(u)=0 LINE TO (1,1) { insulated
right side }
VALUE(u)=1 LINE TO (0,1) { fixed value on
top }
NATURAL(u)=0 LINE TO CLOSE {
insulated left side }
```

Basic steps to solve a problem in FlexPDE

- Define the variables and equations
- Define the domain
- Define the material parameters
- Define the boundary conditions
- Specify the graphical output.

Variables and equations

- what are the variables that you want to analyze?
- what are the partial differential equations that define them?

The **VARIABLES** and **EQUATIONS** sections of a problem script supply this information. The two are closely linked, since you must have one equation for each variable in a properly posed system.

```
VARIABLES
Phi
EQUATIONS
Div(grad(Phi)) = 0
```

```
VARIABLES
A,B
EQUATIONS
A: Div(grad(A)) = 0
B: Div(grad(B)) = 0
```

Mapping the domain: 2D domain description

- A two-dimensional problem domain is described in the **BOUNDARIES** section.
- A **REGION** specification begins with the statement **REGION** <number> (or **REGION** "name") and all loops following the header are included in the region.
- The first **REGION** should contain the entire domain. This is an unenforced convention that makes the attachment of boundary conditions easier.
- Region shapes are described by walking the perimeter, stepping from one joint to another with **LINE**, **SPLINE** or **ARC** segments. <u>Each segment assumes that it will continue from the end of the previous segment</u>, and the **START** clause gets things rolling. You can make a segment return to the beginning with the word **CLOSE** (or **TO CLOSE)**.

Boundary paths

- A boundary path has the general form

  START(a,b) segment TO (c,d) ...

where **(a,b)** and **(c,d)** are the physical coordinates of the ends of the segment, and **segment** is either **LINE**, **SPLINE** or **ARC**.

- The path continues with a connected series of segments, each of which moves the segment to a new point. The end point of one segment becomes the start point of the next segment.
- A path ends whenever the next input item cannot be construed as a segment, or when it is closed by returning to the start point

  ... segment TO CLOSE.

  or

  ... segment CLOSE.

Boundary paths: line and spline segments

- Line segments take the form

  LINE TO (x,y)

  When successive **LINE** segments are used, the reserved word **LINE** does not have to be repeated, as in the following:

  LINE TO (x1,y1) TO (x2,y2) TO (x3,y3) TO ...

- Spline segments are syntactically similar to Line segments

  SPLINE TO (x,y) TO (x2,y2) TO (x3,y3) TO ...

Boundary paths: arc segments

- Arc segments create either circular or elliptical arcs, and take one of the following the forms:

  ARC TO (x1,y1) to (x2,y2)

  ARC (RADIUS = R) to (x,y)

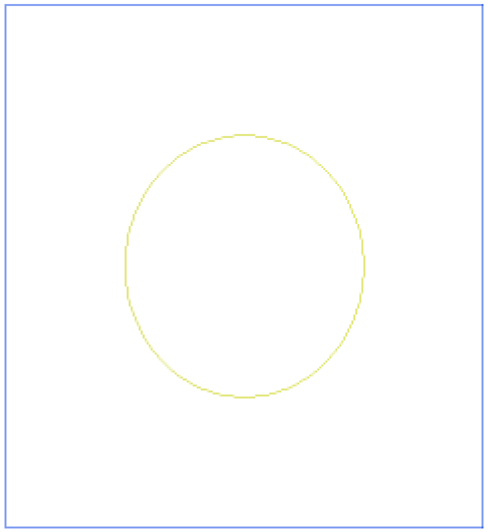
  ARC (CENTER = x1,y1) to (x2,y2)

  ARC (CENTER = x1,y1) ANGLE=angle

- Here **angle** is an angle *measured in degrees*, following the standard convention that positive angles rotate counter-clockwise and negative angles rotate clockwise.
- When the form **ARC (CENTER=x1,y1) to (x2,y2)** is used and the center **(x1,y1)** is not equidistant from the start and end points, an elliptical arc segment is generated with major and minor axes along the X and Y coordinate directions.

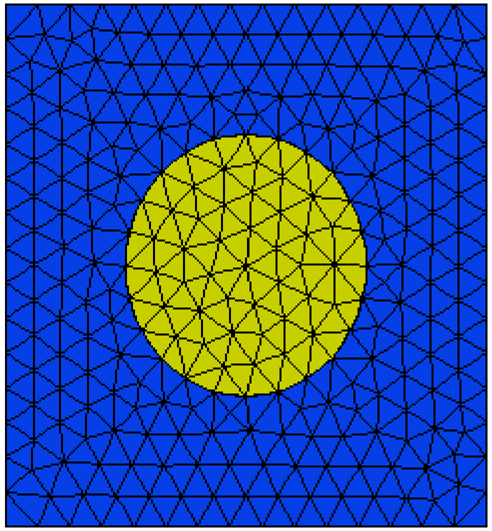
Example of domain

```
DEFINITIONS
R=1/2 {ring radius}
BOUNDARIES
REGION 1 'box' { the bounding box }
START(-1,-1)
LINE TO(1,-1)
TO (1,1)
TO (-1,1)
TO CLOSE
REGION 2 'blob' { the embedded circular 'blob'
    }
START 'ring' (R,0)
ARC(CENTER=0,0) ANGLE=360 TO CLOSE
```



Generating FE mesh

- When you select "Run Script" from the Controls menu (or the button), FlexPDE will begin execution by automatically creating a finite element mesh to fit the domain you have described. In the automatic mesh, cell sizes will be determined by the spacing between explicit points in the domain boundary, by the curvature of arcs, or by explicit user density controls.



Defining material parameters

- This is handled in FlexPDE by two facilities. First, the material parameters are given names and default values in the **DEFINITIONS** section. Second, the material parameters are given regional values within the domain **REGIONS**.

  DEFINITIONS

  k = 3

This default value will be used as the value of "k" in every REGION of the problem, <u>unless</u> specifically redefined in each region, for example:

```
REGION 2 'blob' { the embedded blob }
k = 0.001
START(1/2,0)
ARC(CENTER=0,0) ANGLE=360
```

Defining boundary conditions

- Each boundary condition statement takes as an argument the name of a variable. This name associates the boundary condition with one of the listed equations, for it is in reality the equation that is modified by the boundary condition.

```
REGION 1 'box' { the bounding box }
START(-1,-1)
{ Phi=0 on base line: }
VALUE(Phi)=0 LINE TO(1,-1)
{ normal derivative =0 on right side: }
NATURAL(Phi)=0 LINE TO (1,1)
{ Phi = 1 on top: }
VALUE(Phi)=1 LINE TO (-1,1)
{ normal derivative =0 on left side: }
NATURAL(Phi)=0 LINE TO CLOSE
```

Graphical output

- The **MONITORS** and **PLOTS** sections contain requests for graphical output.
  - **MONITORS** are used to get ongoing information about the progress of the solution.
  - **PLOTS** are used to specify final output, and these graphics will be saved in a disk file for later viewing.
- **Output commands:**

  CONTOUR - a plot of contours of the argument; it may be color-filled

  SURFACE - a 3D surface of the argument

  VECTOR - a field of arrows

  ELEVATION - a "lineout" along a defined path

  SUMMARY - text-only reports

Output commands

- **Plots section**

```
PLOTS
CONTOUR(Phi) {contour plot}
CONTOUR(Phi) painted as “Phi distribution” {filled contour plot with
title}
VECTOR(-k*grad(Phi)) as “Flux distribution” {vector plot with title}
ELEVATION(Phi) FROM (0,-1) to (0,1) {graph along the defined
path}
ELEVATION(Normal(-k*grad(Phi))) ON 'ring'
END
```