Quantum Computing — Exam program

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Each question is implicitly ended with "Examples", i.e. you should provide ones.

Part I. QC Framework

1. Hilbert spaces and Dirac notation. Properties of inner product, dot-product formula, alternatives. Dual vectors, dual spaces. Orthonormal bases. Standard base is orthonormal. Operators, outer products. Theorem: every operator is a linear combination of outer products of base vectors.
2. Adjoint operator: coordinateless and coordinatewise definition, unitary and self-adjoint operators. Unitary operators: definition, intuition, property: real symmetric unitary matrix is self-inverse.
3. Tensor products of spaces (coordinate form) and operators (coordinateless definition). A case of tensor product of vector-columns. Matrix and canonical definition of $A \otimes B$ coincide.
4. A Framework for quantum computations: State Space Postulate, Evolution Postulate, Measurement Postulate.
5. Composition of Systems Postulate, how does it mix with previous postulates. Entangled states, EPR pairs.

Part II. QC Algorithms

6. Superdense coding: task, algorithm, circuit, alternatives for measurement. Holevo's theorem, why it does not contradicts with superdense coding.
7. Quantum teleportation: task, algorithm, circuit. (The implementation using identity: $|\varphi\rangle|b_{00}\rangle = \frac{1}{2}\Sigma_{i,j}\, |b_{ij}\rangle X^j Z^i\, |\varphi\rangle$.) The No-Cloning theorem (NCT): the meaning of cloning in classical sense and in quantum one. Proof. Why NCT does not contradicts with quantum teleportation.
8. Securing CNOT using quantum teleportation.
9. Deutsch and Deutsch—Josza algorithms.
10. Simon's Problem.
11. Phase estimation problem and Quantum Fourier Transformation.
12. Eigenvalue estimation. The issue of precision in QFT^{-1}, probability for getting the best approximation.
13. Reduction of Order-Finding Problem to Eigenvalue Estimation Problem. Problems and their resolution.