Интерполяционный многочлен Ньютона

$$\boldsymbol{f(x) = x^3 + 1}$$

$x_0 = 0$	$f_0 = 1$	$f(x_0; x_1) = \frac{f_1 - f_0}{x_1 - x_0} = 1$	$f(x_0; x_1; x_2) = \frac{f(x_1; x_2) - f(x_0; x_1)}{x_2 - x_0} = \frac{7-1}{2-0} = 3$	$f(x_0; x_1; x_2; x_3) = \frac{6-3}{3} = 1$
$x_1 = 1$	$f_1 = 2$	$f(x_1; x_2) = \frac{f_2 - f_1}{x_2 - x_1} = 7$	$f(x_1; x_2; x_3) = \frac{19-7}{3-1} = 6$	
$x_2 = 2$	$f_2 = 9$	$f(x_2; x_3) = 19$		
$x_3 = 3$	$f_3 = 28$			

$$N_3(x) = f(x_0) + f(x_0; x_1)(x - x_0) + f(x_0; x_1; x_2)(x - x_0)(x - x_1) + f(x_0; x_1; x_2; x_3)(x - x_0)(x - x_1)(x - x_2) = 1 + 1(x - 0) + 3(x - 0)(x - 1) + 1(x - 0)(x - 1)(x - 2) =$$

$$N_3(x) = 1 + x + 3x^2 - 3x + x^3 - 3x^2 + 2x = x^3 + 1$$

Многочлен Эрмита

$\boldsymbol{x_1 = 1}$	$\boldsymbol{x_2 = 2}$	$\boldsymbol{x_3 = 3}$
$\boldsymbol{f_1 = 2}$	$\boldsymbol{f_2 = 9}$	$\boldsymbol{f_3 = 28}$
$\boldsymbol{df_1 = 3}$		

Решение

$\boldsymbol{x_0^\varepsilon = x_1 + \varepsilon}$	$\boldsymbol{x_1 = 1}$	$\boldsymbol{x_2 = 2}$	$\boldsymbol{x_3 = 3}$
$\boldsymbol{f_0^\varepsilon}$	$\boldsymbol{f_1 = 2}$	$\boldsymbol{f_2 = 9}$	$\boldsymbol{f_3 = 28}$
	$\boldsymbol{df_1 = 3}$		

$x_0 = 1 + \varepsilon$	f_0^ε	$f(x_0^\varepsilon; x_1) = \frac{f_1 - f_0^\varepsilon}{x_1 - x_0^\varepsilon} = \frac{f(x_1) - f(x_1 + \varepsilon)}{\varepsilon} = df1 = 3$	$f(x_0^\varepsilon; x_1; x_2) = \frac{f(x_1; x_2) - f(x_0^\varepsilon; x_1)}{x_2 - x_0^\varepsilon} = \frac{7-3}{2-1} = 4$	$f(x_0^\varepsilon; x_1; x_2; x_3) = \frac{6-4}{2} = 1$
$x_1 = 1$	$f_1 = 2$	$f(x_1; x_2) = \frac{f_2 - f_1}{x_2 - x_1} = 7$	$f(x_1; x_2; x_3) = \frac{19-7}{3-1} = 6$	
$x_2 = 2$	$f_2 = 9$	$f(x_2; x_3) = 19$		
$x_3 = 3$	$f_3 = 28$			

$$N_3(x) = f(x_0^\varepsilon) + f(x_0^\varepsilon; x_1)(x - x_0^\varepsilon) + f(x_0^\varepsilon; x_1; x_2)(x - x_0^\varepsilon)(x - x_1) + f(x_0^\varepsilon; x_1; x_2; x_3)(x - x_0^\varepsilon)(x - x_1)(x - x_2)$$

$$\boldsymbol{N_3(x) = f_1 + f(x_1; x_1)(x - x_1) + f(x_1; x_1; x_2)(x - x_1)^2 + f(x_1; x_1; x_2; x_3)(x - x_1)^2(x - x_2) =}$$

$$\boldsymbol{N_3(x) = 2 + 3(x - 1) + 4(x - 1)^2 + 1(x - 1)^2(x - 2) = 2 + 3x - 3 + 4x^2 - 8x + 4 + (x^2 - 2x + 1)(x - 2) = x^3 + 1}$$