Algorithms and Data Structures

Module 1

Lecture 3

Graphs: definitions, representations and basic operations

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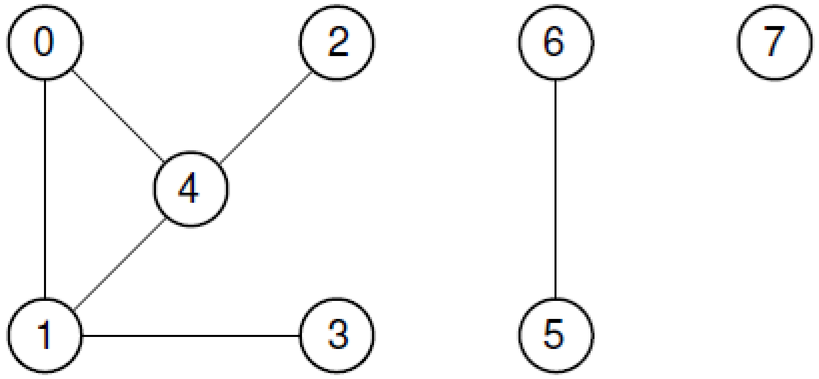
Graphs: definition

Graph G=(V,E)

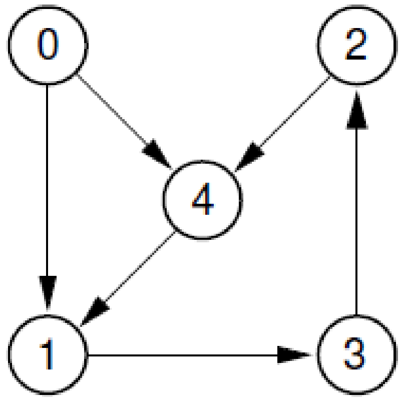
✓ *V* is a set of ***vertices*** ($v \in V$ – vertex, node). |*V*|=*n*.

✓ *E* is a set of ***edges*** ($e = (v, w)$: $v, w \in V$ – edge, arc). |*E*|=*m*

Undirected graph



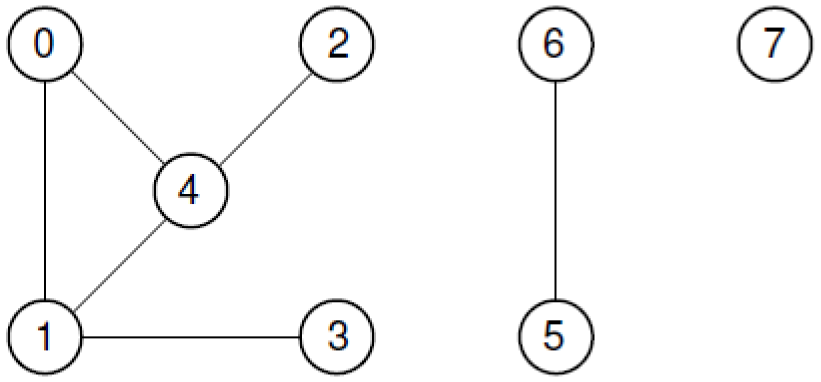
Directed graph

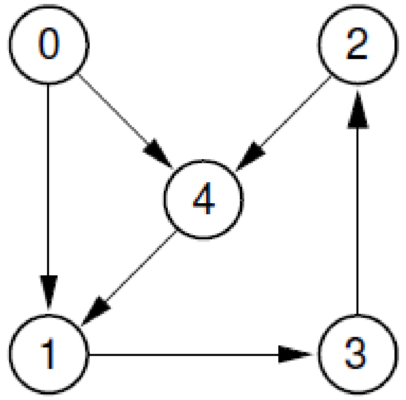


Graphs: definition

$e = (v, w): v, w \in V$

- ✓ e is ***incident*** to v and w ; v (w) is incident to e;
- ✓ v and w are ***adjacent***; they are *neighbours*.

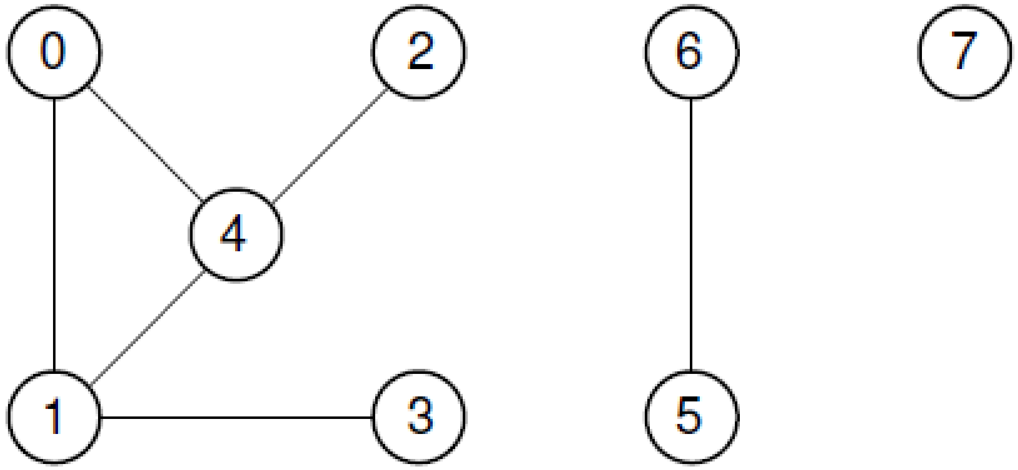




Graphs: definition

$v \in V$:

✓ $\deg(v)$ – ***degree*** of vertex v = number of edges incident to v .



Graphs: definition

$v \in V$:

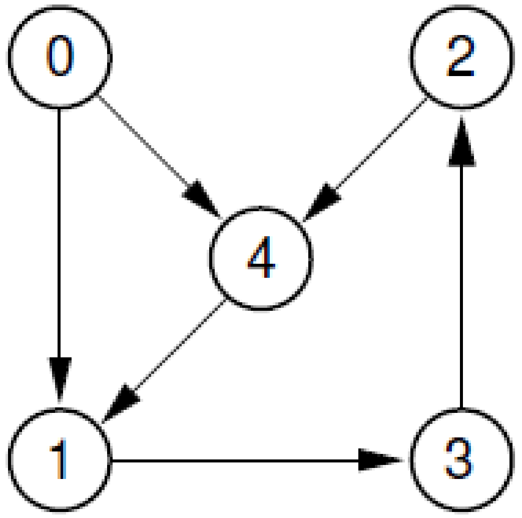
✓ $\deg(v)$ – ***degree*** of vertex v = number of edges incident to v .

✓ $\text{outdeg}(v)$– out-degree of vertex v = number of edges which start from v .

✓ $\text{indeg}(v)$– in-degree of vertex v = number of edges which end at v .

✓ v is a ***source*** iff $\text{indeg}(v) = 0$

✓ v is a ***sink*** iff $\text{outdeg}(v) = 0$

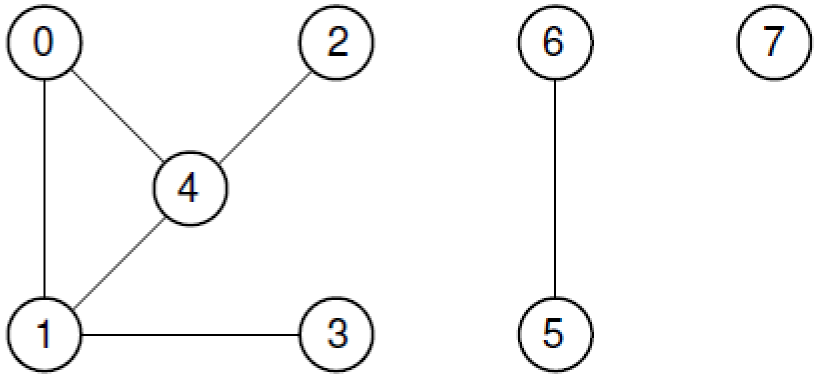


Graphs: representations

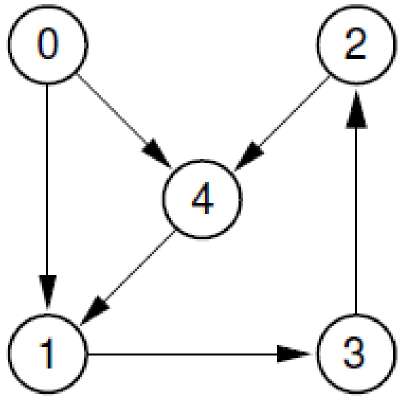
Edge list

$$E = \{e_1 = (u_1, v_1), \ldots, e_m = (u_m, v_m)\}$$

0	1
0	4
1	3
1	4
2	4
5	6



0	1
0	4
1	3
4	1
2	4
3	2



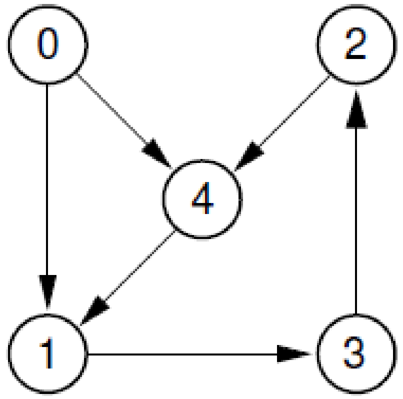
Graphs: representations

Edge list

$$E = \{e_1 = (u_1, v_1), \dots, e_m = (u_m, v_m)\}$$

Property	Complexity
out-deg(v)	$O(m)$
in-deg(v)	$O(m)$
deg(v)	$O(m)$
has_edge(v,w)	$O(m)$
is_source(v)	$O(m)$
is_sink(v)	$O(m)$

0	1
0	4
1	3
4	1
2	4
3	1

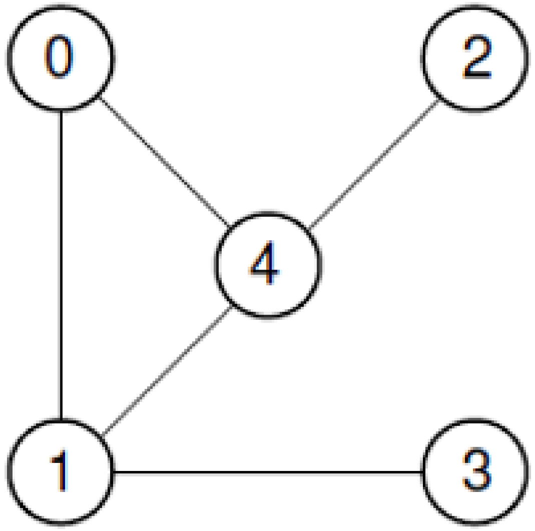


Graphs: representations

Adjacency matrix

$$A = \{a_{ij}\}_{i,j=1}^{n} : a_{ij} = \begin{cases} 1, if\ (i,j) \in E \\ 0,\ otherwise \end{cases}$$

$$\begin{array}{c|ccccc|} & 0 & 1 & 2 & 3 & 4 \\ \hline 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 2 & 0 & 0 & 0 & 0 & 1 \\ 3 & 0 & 1 & 0 & 0 & 0 \\ 4 & 1 & 1 & 1 & 0 & 0 \\ \hline \end{array}$$

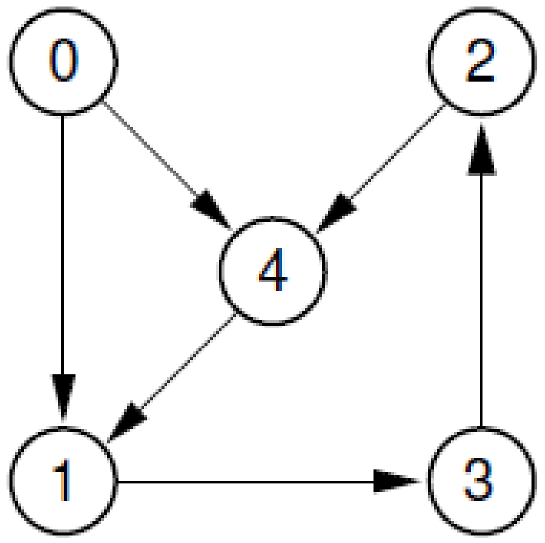


Graphs: representations

Adjacency matrix

$$A = \{a_{ij}\}_{i,j=1}^{n}: a_{ij} = \begin{cases} 1, if\ (i,j) \in E \\ 0,\ otherwise \end{cases}$$

$$\begin{array}{c|ccccc|}
 & 0 & 1 & 2 & 3 & 4 \\
\hline
0 & 0 & 1 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 1 & 0 \\
2 & 0 & 0 & 0 & 0 & 1 \\
3 & 0 & 0 & 1 & 0 & 0 \\
4 & 0 & 1 & 0 & 0 & 0 \\
\hline
\end{array}$$



Graphs: representations

Adjacency matrix

$A = \{a_{ij}\}_{i,j=1}^{n}$ contains $O(n^2)$ entries.

- Space-efficient for ***dense*** graphs

$(m \sim O(n^2))$.

- Space-inefficient for ***sparse*** graphs

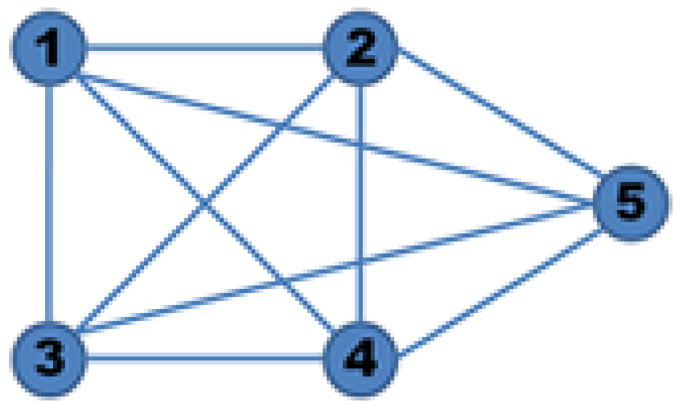
$(m \sim O(n))$.

$$\begin{array}{c|ccccc|}
 & 0 & 1 & 2 & 3 & 4 \\
\hline
0 & 0 & 1 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 1 & 0 \\
2 & 0 & 0 & 0 & 0 & 1 \\
3 & 0 & 0 & 1 & 0 & 0 \\
4 & 0 & 1 & 0 & 0 & 0 \\
\hline
\end{array}$$

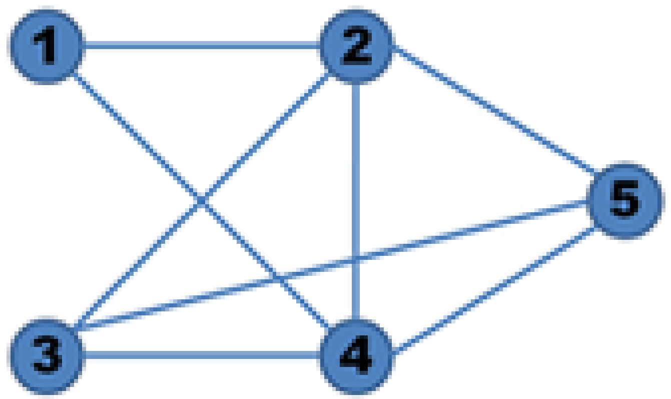
Graphs: representations

Adjacency matrix

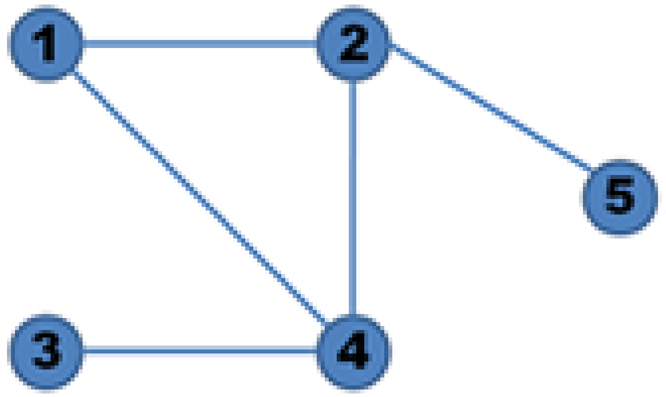
Complete Graph



Dense Graph



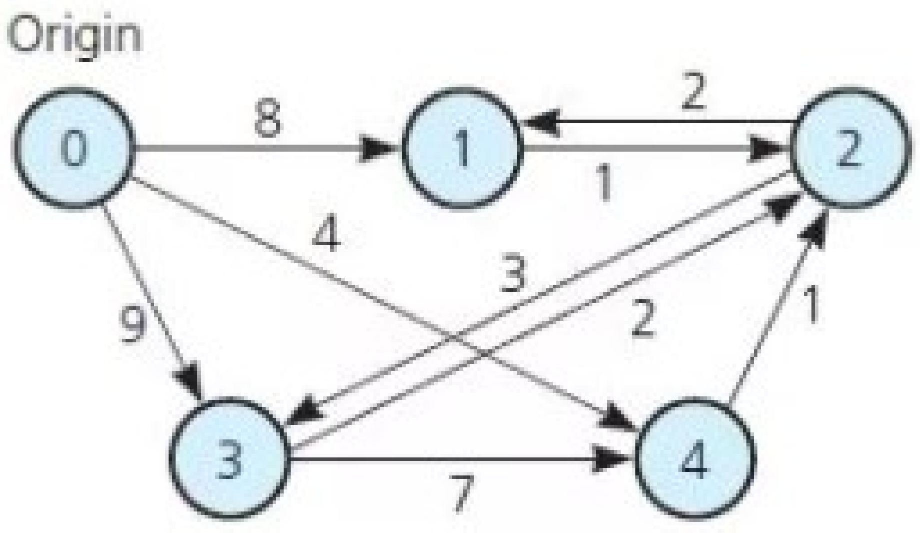
Sparse Graph



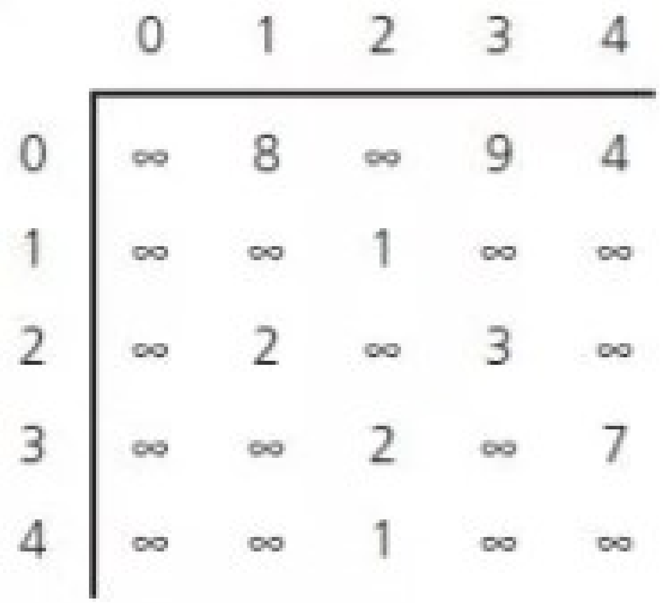
Graphs: representations

Adjacency matrix

(a)



(b)



	0	1	2	3	4
0	∞	8	∞	9	4
1	∞	∞	1	∞	∞
2	∞	2	∞	3	∞
3	∞	∞	2	∞	7
4	∞	∞	1	∞	∞

Graphs: representations

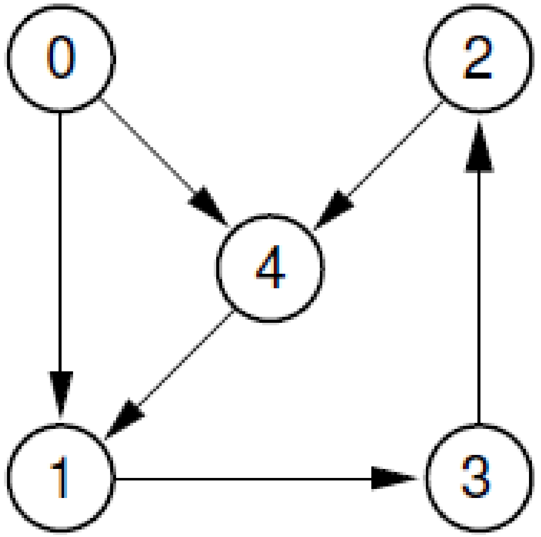
Adjacency matrix

Property	Complexity
out-deg(v)	$O(n)$
in-deg(v)	$O(n)$
deg(v)	$O(n)$
has_edge(v,w)	$O(1)$
is_source(v)	$O(n)$
is_sink(v)	$O(n)$

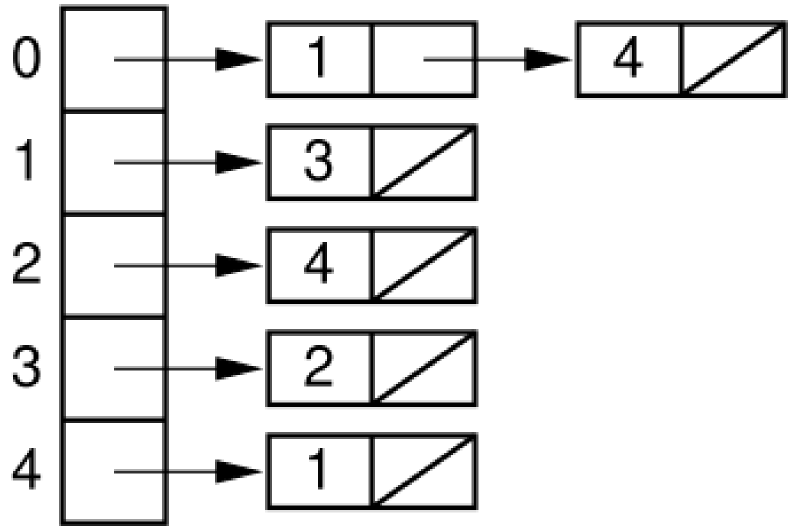
$$\begin{array}{c|ccccc|}
 & 0 & 1 & 2 & 3 & 4 \\
\hline
0 & 0 & 1 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 1 & 0 \\
2 & 0 & 0 & 0 & 0 & 1 \\
3 & 0 & 0 & 1 & 0 & 0 \\
4 & 0 & 1 & 0 & 0 & 0 \\
\hline
\end{array}$$

Graphs: representations

Adjacency list



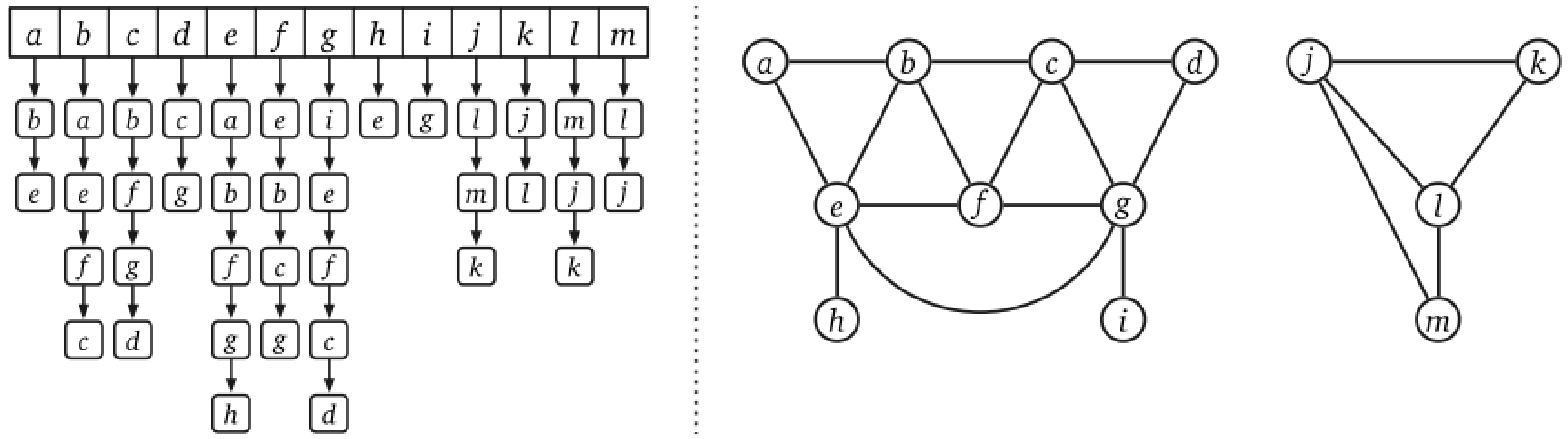
$$
\begin{array}{c|ccccc|}
 & 0 & 1 & 2 & 3 & 4 \\
\hline
0 & 0 & 1 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 1 & 0 \\
2 & 0 & 0 & 0 & 0 & 1 \\
3 & 0 & 0 & 1 & 0 & 0 \\
4 & 0 & 1 & 0 & 0 & 0 \\
\hline
\end{array}
$$



Space complexity: $O(n+m)$

Graphs: representations

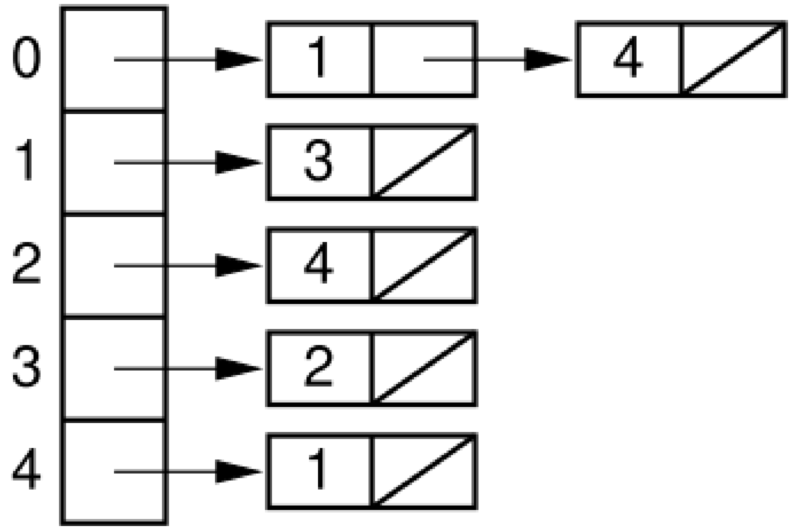
Adjacency list



Graphs: representations

Adjacency list

Property	Complexity
out-deg(v)	$O(\max deg) = O(n)$
in-deg(v)	$O(n+m) = O(m)$
deg(v)	$O(m)$
has_edge(v,w)	$O(\max deg) = O(n)$
is_source(v)	$O(m)$
is_sink(v)	$O(1)$



Space complexity: $O(n+m)$

Graphs: representations

Property	Edge list	Adjacency matrix	Adjacency list
out-deg(v)	$O(m)$	$O(n)$	$O(\max deg) = O(n)$
in-deg(v)	$O(m)$	$O(n)$	$O(n + m) = O(m)$
deg(v)	$O(m)$	$O(n)$	$O(m)$
has_edge(v,w)	$O(m)$	$O(1)$	$O(\max deg) = O(n)$
is_source(v)	$O(m)$	$O(n)$	$O(m)$
is_sink(v)	$O(m)$	$O(n)$	$O(1)$
memory	$O(m)$	$O(n^2)$	$O(n + m)$