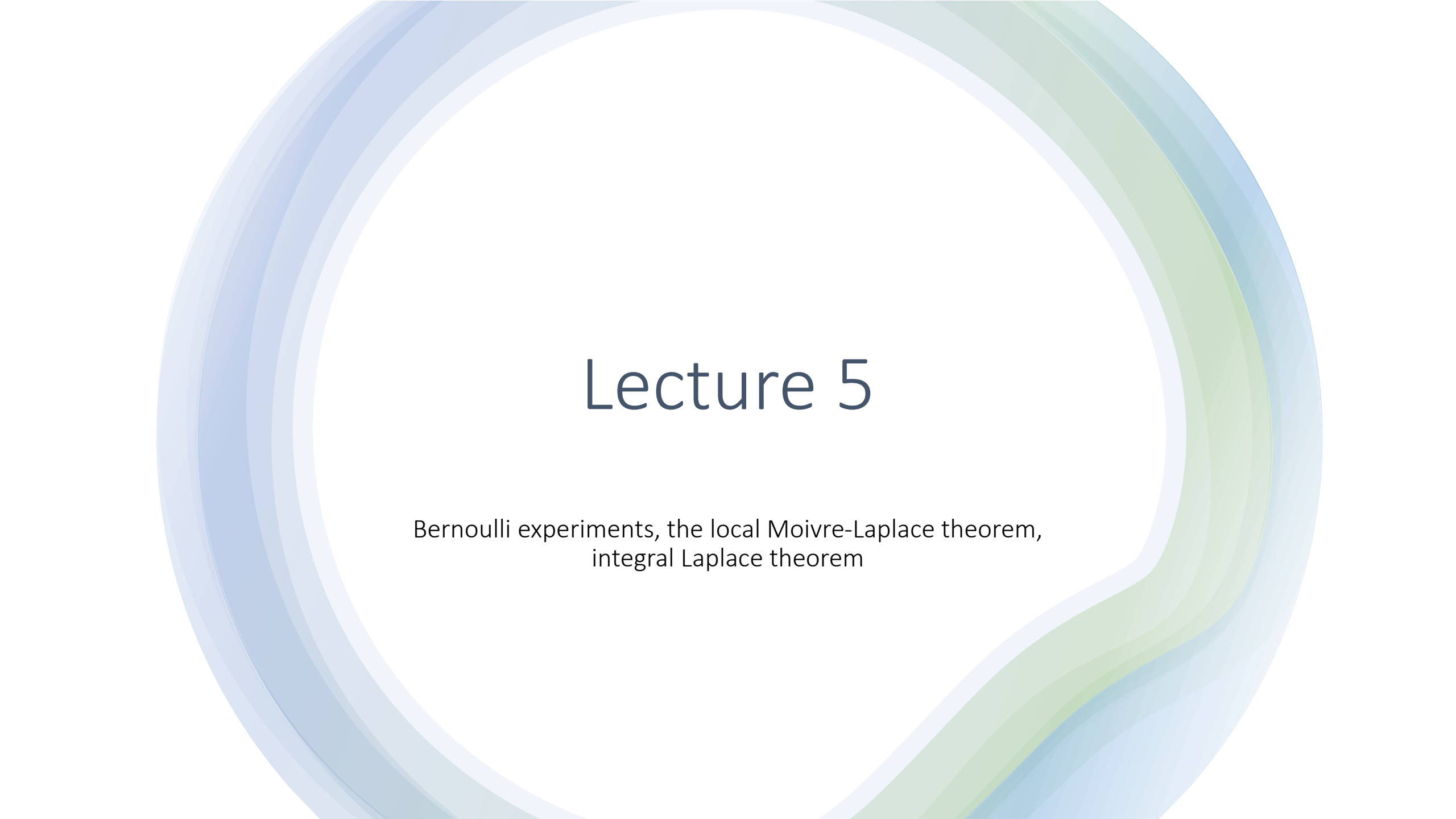




Lecture 5

Bernoulli experiments, the local Moivre-Laplace theorem,
integral Laplace theorem

The local Moivre-Laplace theorem

Let n be big enough.

Theorem. If the probability p of the occurrence of the event A in each trial is constant and different from zero and one, then the probability $P_n(k)$ that the event A in n trials will occur k times is approximately equal to the value of the function $y = \frac{1}{\sqrt{npq}} \varphi(x)$ where $x = \frac{k-np}{\sqrt{npq}}$, and $\varphi(x)$ – Gaussian function.

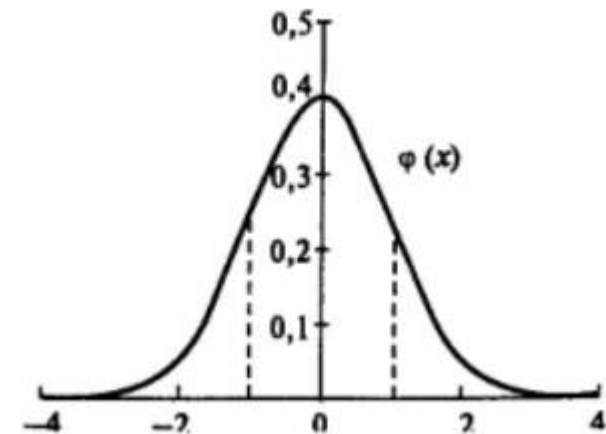
$$\varphi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

Properties:

1. $\varphi(x) > 0$
2. $\varphi(-x) = \varphi(x)$

That is, the probability $P_n(k)$ that the event A in n trials will occur k times is approximately equal to $P_n(k) \approx \frac{1}{\sqrt{npq}} \varphi(x)$

where $x = \frac{k-np}{\sqrt{npq}}$



The local Moivre-Laplace theorem

Example. A die is tossed 500 times. What is the probability that the number 1 will appear 50 times?

Integral Laplace theorem

Let n be big enough.

Theorem. If the probability p of the occurrence of the event A in each trial is constant and different from zero and one, then the probability $P_n(k_1, k_2)$ that event A in n trials will come from k_1 to k_2 times is approximately equal to a definite integral

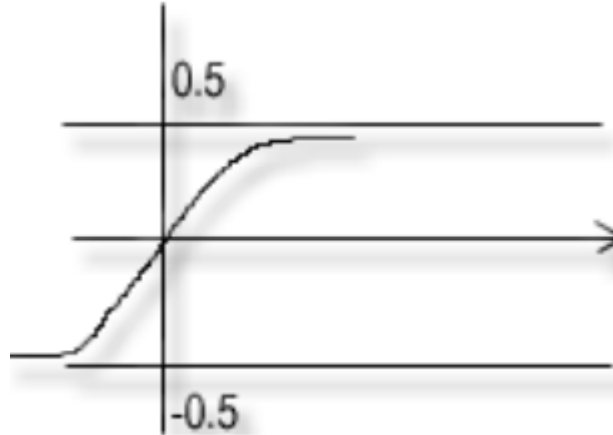
$$P_n(k_1, k_2) = \frac{1}{\sqrt{2\pi}} \int_{x'}^{x''} e^{-\frac{z^2}{2}} dz, \text{ где } x' = \frac{k_1 - np}{\sqrt{npq}}, x'' = \frac{k_2 - np}{\sqrt{npq}}$$

$$\begin{aligned} \text{Laplace function } \Phi(x) &= \frac{1}{\sqrt{2\pi}} \int_0^x e^{-\frac{z^2}{2}} dz, \text{ then } P_n(k_1, k_2) = \frac{1}{\sqrt{2\pi}} \int_{x'}^{x''} e^{-\frac{z^2}{2}} dz = \frac{1}{\sqrt{2\pi}} \int_{x'}^0 e^{-\frac{z^2}{2}} dz + \\ &+ \frac{1}{\sqrt{2\pi}} \int_0^{x''} e^{-\frac{z^2}{2}} dz = \frac{1}{\sqrt{2\pi}} \int_0^{x''} e^{-\frac{z^2}{2}} dz - \frac{1}{\sqrt{2\pi}} \int_0^{x'} e^{-\frac{z^2}{2}} dz = \Phi(x'') - \Phi(x') \end{aligned}$$

Integral Laplace theorem

Properties of the Laplace function:

1. Even function
2. $-0.5 \leq \Phi(x) \leq 0.5$



$$P_n(k_1, k_2) \approx \Phi(x'') - \Phi(x'), \text{ где } x' = \frac{k_1 - np}{\sqrt{npq}}, x'' = \frac{k_2 - np}{\sqrt{npq}}$$

Integral Laplace theorem

Example. Find the probability that if you toss a coin 200 times, it will come up heads 90 to 110 times.

Tasks

Tasks

1. Find the probability that event A will occur 1400 times in 2400 trials if the probability of this event occurring in each trial is 0.6.
2. The probability of hitting a target with one shot is 0.8. Find the probability that with 100 shots the target will be hit exactly 75 times.
3. A student will receive a credit if they answer 25 or more questions of the test correctly out of 40. What is the probability of getting a credit if the student answers simply at random "true" - "false"?
4. The probability of an event occurring in each of 100 independent trials is constant and equal to 0.8. Find the probability that the event will occur: a) at least 75 times and no more than 90 times; b) at least 75 times; c) no more than 74 times.

Tasks

5. The probability of an event occurring in each of the independent trials is 0.8. How many trials must be performed in order to expect the event to occur at least 75 times with a probability of 0.9?

Preparing for the test

1. There are 20 candies in the package, 15 of which are chocolate, and 5 are lollipops. The girl randomly takes out 7 sweets. Find the probability that all the extracted candies are chocolate.
2. When studying the authorized capital of banks, it was found that a fifth of the banks have an authorized fund of more than 100 million rubles. rub. Find the probability that among 1800 banks have an authorized capital of more than 100 million. rub.:
 - a) not less than 300;
 - b) from 300 to 400 inclusive
3. Suppose the code consists of 2 letters followed by 3 digits. Find the number of:
 - a) codes
 - b) codes with different letters
 - c) codes with the same letters.

Preparing for the test

4. The student knows 18 of the 23 questions in the program. Find the probability that the student knows the 3 questions asked by the examiner.
5. Ten students are standing in a queue in a library. How many possible queues are there? How many possible queues will there be in which:
 - a) three specific students A, B, and C stand next to each other in the sequence ABC;
 - b) three specific students A, B, and C stand next to each other?
6. Three shooters shoot at the same target simultaneously. The probabilities of hitting with one shot are 0.7, 0.8, and 0.9, respectively. Find the probability that with a simultaneous volley of these shooters the targets will be:
 - a) only one hit;
 - b) at least one hit.

Preparing for the test

7. The test on the subject consists of six problems, and the test is considered passed if the student has solved at least four of them. The student can solve each problem with a probability of 0.6. What is the probability that he will pass the test?
8. Calculator manufacturers know from experience that 1% of calculators sold are defective. An audit firm bought 500 calculators. What is the probability that 4 calculators will have to be replaced? No more than 20 calculators?
9. There are 28 tiles in a game of dominoes. Dominoes are played by four people who, when starting the game, take all the tiles apart. How many different ways of taking apart the tiles in a game of dominoes are possible?
10. Two different digits are randomly selected from the digits 1 through 5. What is the probability that 2 is one of the selected digits if the sum of the digits is odd? What is the probability that the sum of the digits is odd if 2 is one of the digits?

Preparing for the test

11. Four students from the first group, six from the second, and five from the third are selected to participate in the student selection sports competition. The probabilities of a student from each group getting into the university team are 0.5; 0.4; 0.3, respectively. What is the probability that a randomly selected participant in the competition got into the team? A randomly selected student got into the team. What is the probability that he is from the third group.

12. A dental clinic distributes advertising leaflets at the entrance to the metro. Experience shows that in one case out of a thousand, a visit to the clinic follows. Find the probability that when distributing 50 thousand leaflets, the number of visits will be:

- a) equal to 41,
- b) be between 36 and 47

Homework

1. Four customers arrived at the warehouse. The probability that each of these customers will need a refrigerator is 0.4. Find the probability that a refrigerator will be needed by: a) at least two customers; b) at most three customers; c) all four customers.
2. Find the probability that event A occurs exactly 70 times in 243 trials if the probability of this event occurring in each trial is 0.25.
3. Statistics from company audits state that the probability of detecting an error in each document being audited is 0.1. What is the probability that out of ten documents being audited, nine of them will be error-free?
4. A student will receive a pass if he answers 30 or more questions out of 50 correctly. What is the probability of receiving a pass if a student simply answers “true” – “false” at random?
5. The probability of failure of one machine is 0.12. Determine the probability of failure of 4 to 10 machines if there are 155 machines.