| **Basic distributions and properties** | | |
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| **Discrete distributions** | Formation of a random variable (r.v.) | Examples of real features having this distribution |
| **Bernoulli**<br>$\mathrm{P}(X=x)=p^x(1-p)^{1-x}, \quad x=0,1; \quad 0\le p\le 1$<br>$\mathbb{E}(X)=p$<br>$\mathrm{var}(X)=p(1-p)$ | | |
| **Binomial**<br>$\mathrm{P}(X=x)=\binom{n}{x}p^x(1-p)^{n-x}, \quad x=0,1,...,n; \quad 0\le p\le 1$<br>$\mathbb{E}(X)=np$<br>$\mathrm{var}(X)=np(1-p)$ | r.v. – the appearance of success in a sequence of n independent trials;<br><br>p – is the probability of success in the outcome of the experiment | 1) the number n of defective products in a batch of volume produced in stationary mode;<br><br>2) the number of objects with a given combination of properties they are among n randomly selected from the general population |
| **Geometric**<br>$\mathrm{P}(X=x)=p(1-p)^{x-1}, \quad x=1,2,...; \quad 0\le p\le 1$<br>$\mathbb{E}(X)=\frac{1}{p}$<br>$\mathrm{var}(X)=\frac{1-p}{p^2}$ | this distribution allows us to find the probability that the first occurrence of success requires $x$ independent trials, each with a probability of success p. | 1) the probability that success will not follow x times (medicine will not help, a specific value on the cube will not fall out, etc.) |
| **Multinomial**<br>$\mathrm{P}(X_1=x_1,X_2=x_2,...,X_m=x_m)=\frac{n!}{x_1!x_2!\cdots x_m!}p_1^{x_1}p_2^{x_2}\cdots p_m^{x_m}$<br>$x_1+\cdots+x_m=n;$<br>$p_1+\cdots+p_m=1$<br>$\mathbb{E}(X_i)=p_i$<br>$\mathrm{var}(X_i)=np_i(1-p_i)$<br>$\mathrm{cov}(X_i,X_j)=-np_ip_j$ | The number of $x_1, x_{2,...}, x_m$ occurrences of interesting, mutually incompatible events $A_1, A_{2,...}, A_m$, forming a complete system of events<br>**An example?** | Polynomial:<br>A set of numbers that defines the distribution of n objects by m specified properties when they are independently extracted from the general population, while the probability of an event $A_j$ is constant and equal to $p_j$ |

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| **Negative Binomial**<br>$\mathrm{P}(X=x)=\dfrac{\Gamma(r+x)}{x!\Gamma(r)}p^r(1-p)^{x-1}, \quad x=0,1,2,...; \quad 0\le p\le 1$<br>$\mathbb{E}(X)=\dfrac{r(1-p)}{p}, \quad \mathrm{var}(X)=\dfrac{r(1-p)}{p^2}$ | r.v. – the number of tests of independent experiments that had to be carried out in anticipation of a given number $r$ of the occurrence of the event of interest to us, $p$ – the probability of success in one test. | 1) The durability of the system (in the number of cycles of operation), having $r$-1 backup (automatically connected) elements<br><br>2) The sample size required to obtain $r$ objects with the specified properties when they are randomly extracted from the general population |
| **Poisson**<br>$\mathrm{P}(X=x)=\dfrac{\exp(-\lambda)\lambda^x}{x!}, \quad x=0,1,2,..., \quad \lambda>0$<br>$\mathbb{E}(X)=\lambda$<br>$\mathrm{var}(X)=\lambda$ | (rare events)<br>r.v. – the number of occurrence of an event per unit of time in stationary mode (previous events do not affect subsequent ones!) | 1) the number of failures of debugged production in unit time<br><br>2) The number of accidents or deaths from rare diseases per unit of time in this population<br><br>3) the number of requests received per unit of time in the queuing system (no longer!) |
| **Continuous distributions** | | |
| **Beta**<br>$f(x)=\dfrac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}x^{\alpha-1}(1-x)^{\beta-1}, \quad 0\le x\le 1; \quad \alpha>0, \ \beta>0$<br>$\mathbb{E}(X)=\dfrac{\alpha}{\alpha+\beta}, \quad \mathrm{var}(X)=\dfrac{\alpha\beta}{(\alpha+\beta+1)(\alpha+\beta)^2}$ | | a wide class of investigated features, the possible values of which are enclosed in the interval (0,1), for example, by expert means certain probabilities of the event of interest to us |
| **Cauchy**<br>$f(x)=\dfrac{1}{\pi(1+x^2)}, \quad -\infty<x<\infty$<br>$\mathbb{E}(X)$ not defined<br>$\mathrm{var}(X)=\infty$ | | The ratio of two independent normal random variables with zero mean and unit variance |

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| **Exponential**<br>$f(x)=\frac{1}{\theta}\exp\left(\frac{x}{\theta}\right), \quad 0\le x<\infty; \quad \theta>0$<br>$\mathbb{E}X=\theta\,, \qquad \mathrm{var}(X)=\theta^2$ | A random time interval between two Poisson-type events | 1) device maintenance time<br><br>2) product durability (normal operation mode)<br><br>3) the duration of the technological operation<br><br>4) the time interval between consecutive failures |
| **Logistic**<br>$f(x)=\frac{\exp(-x)}{(1+\exp(-x))^2}, \quad -\infty<x<\infty;$<br>$\mathbb{E}(X)=0\,, \quad \mathrm{var}(X)=\frac{\pi^2}{3}$ | artificial distribution | |
| **Lognormal**<br>$f(x)=\frac{1}{\sqrt{2\pi}\sigma x}\exp\left(-\frac{(\log x-\mu)^2}{2\sigma^2}\right), \quad 0\le x<\infty; \quad \sigma>0$<br>$\mathbb{E}(X)=\exp(\mu+\sigma^2/2)$<br>$\mathrm{var}(X)=\exp(2\mu+2\sigma^2)-\exp(2\mu+\sigma^2)$ | r.v.is formed under the influence of a large number of independent factors, each of which does not prevail over the others, but the nature of the impact is multiplicative, random growth is due to the action of each new factor | 1) wages in the aggregate of all employees<br><br>2) average per capita income in a set of families of a given size<br><br>3) particle size and volume during crushing |
| **Pareto**<br>$f(x)=\frac{\beta\alpha^\beta}{x^{\beta+1}}, \quad \alpha\le x<\infty, \quad \alpha>0, \quad \beta>0$<br>$\mathbb{E}(X)=\frac{\beta\alpha}{\beta-1}, \quad \beta>1$<br>$\mathrm{var}(X)=\frac{\beta\alpha^2}{(\beta-1)^2(\beta-2)}, \qquad \beta>2$ | The nature of the variation of the studied feature from general population from which all elements, that do not exceed the threshold are extracted in advance $\alpha$ | Distribution of the average per capita total income of families with an income of at least a given $\alpha$ |

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| **Uniform**<br>$f(x) = \frac{1}{b-a}, \qquad a \le x \le b$<br>$\mathbb{E}(X) = \frac{a+b}{2}, \quad \mathrm{var}(X) = \frac{(b-a)^2}{12}$ | r.v. is formed in such a way that the probability of getting an observation in the vicinity of a given range [a,b] depends only on its width | 1) rounding error of calculations with a fixed number of decimal places<br><br>2) waiting time for maintenance with exactly periodic activation (arrival) of the device (application) |
| **Weibull**<br>$f(x) = \frac{\gamma}{\beta} x^{\gamma-1} \exp\left(-\frac{x^\gamma}{\beta}\right), \qquad 0 \le x < \infty; \qquad \gamma > 0,\ \beta > 0$<br>$\mathbb{E}(X) = \beta^{1/\gamma}\Gamma\left(1+\frac{1}{\gamma}\right)$<br>$\mathrm{var}(X) = \beta^{2/\gamma}\left(\Gamma\left(1+\frac{2}{\gamma}\right) - \Gamma^2\left(1+\frac{1}{\gamma}\right)\right)$ | 1) durability tests in the case when the failure rate function (the "mortality" coefficient) belongs to the class of power dependencies<br><br>2) the distributions of the smallest values in large samples extracted from general population (bounded on the left) are described. | 1) The durability of the system or product operating in the run-in mode $\gamma \in (0,1)$, normal operation $\gamma=1$ or wear and aging $\gamma>1$<br><br>2) the number of cycles until the moment of destruction of the sample<br><br>3) A minimum (for a number of years) of a certain amount consumed per year (for example, water during droughts) |
| **Gamma**<br>$f(x) = \frac{1}{\Gamma(\alpha)\theta^\alpha} x^{\alpha-1} \exp\left(-\frac{x}{\theta}\right), \quad 0 \le x < \infty; \quad \alpha > 0,\ \theta > 0$<br>$\mathbb{E}(X) = \alpha\theta, \qquad \mathrm{var}(X) = \alpha\theta^2$ | theoretical distribution | |
| **Chi-Square**<br>$f(x) = \frac{1}{\Gamma(r/2)2^{r/2}} x^{r/2-1} \exp\left(-\frac{x}{2}\right), \quad 0 \le x < \infty; \quad r > 0$<br>$\mathbb{E}(X) = r, \quad \mathrm{var}(X) = 2r$ | by definition | 1) estimation of variance (normalized)<br><br>2) measure of deviation of the empirical distribution from the hypothetical one |
| **Normal**<br>$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right), \quad -\infty < x < \infty; \quad -\infty < \mu < \infty,\ \sigma^2$<br>$\mathbb{E}(X) = \mu, \qquad \mathrm{var}(X) = \sigma^2$ | r.v. is formed under the influence of a large number of independent factors, each of which does not prevail over the others | 1) deviation from the nominal value in the values of the parameters of products of mass stationary production<br><br>2) measurement error<br><br>3) error when shooting at the target |

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| **Student t** <br> $$f(x)=\frac{\Gamma\left(\frac{r+1}{2}\right)}{\sqrt{r\pi}\,\Gamma\left(\frac{r}{2}\right)}\left(1+\frac{x^2}{r}\right)^{-\left(\frac{r+1}{2}\right)}, \quad -\infty<x<\infty; \quad r>0$$ <br> $\mathbb{E}(X)=0$ if $r>1$, $\quad \mathrm{var}(X)=\frac{r}{r-2}$ if $r>2$ | by definition | 1) a normalized measure of the discrepancy between two sample averages <br><br> 2) deviation of the sample average from the hypothetical one |