Lecture 8

Discrete random variables (tasks), continuous random variables

Tasks

1. The distribution of the random variable X has the form:

X	-2	1	2	5	7
p	0.3	0.1	0.1	0.3	0.2

Find its mathematical expectation, variance and standard deviation. Find the CDF of the random variable X and plot it.

2. Discrete random variable X takes three possible values : $x_1 = 4$ with a probability $p_1 = 0.5$; $x_2 = 6$ with a probability $p_2 = 0.3$ and x_3 with a probability p_3. Find x_3 and p_3 knowing that E(X) = 8.

3. Find the variance of a discrete random variable X—the number of occurrences of event A in five independent trials, if the probability of occurrence of events A in each trial is 0.2.

Continuous random variable, PDF

Consider random variables with a continuous range of possible values.

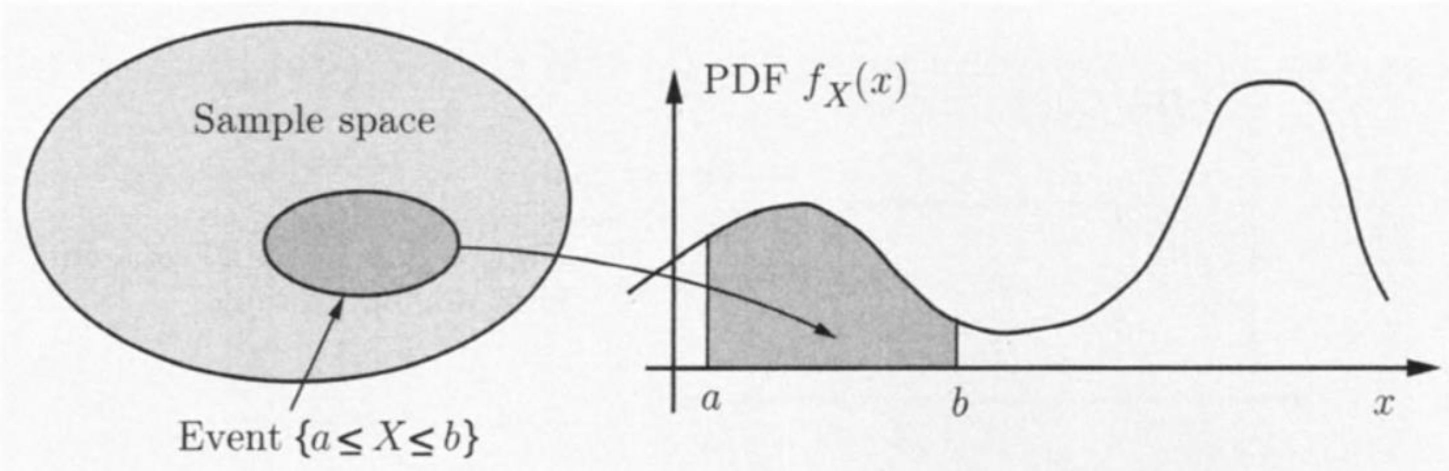
A random variable *X* is called **continuous** if there is a nonnegative function f, called **the probability density function** of *X* ***(PDF)*** such that:

$$P(X \in B) = \int_B f(x)dx$$

for every subset *B* of the real line.

In particular, the probability that the value of *X* falls within an interval is

$$P(a \le X \le B) = \int_a^b f(x)dx$$



Continuous random variable, PDF

1. $f(x) \geq 0$
2. $\int_{-\infty}^{+\infty} f(x)dx = 1$

Expectation: $E(X) = \int_{-\infty}^{+\infty} xf(x)dx$

Variance: $var(X) = E(X^2) - (E(X))^2$, where $E(X^2) = \int_{-\infty}^{+\infty} x^2 f(x)dx$

Standard deviation: $\sigma = \sqrt{var(X)}$

Cumulative distribution function

$F(x) = P(X < x) = P(-\infty < X < x) = \int_{-\infty}^{x} f(t)dt$, that is:

$$\boldsymbol{F(x) = \int_{-\infty}^{x} f(t)dt}$$

$$\boldsymbol{f(x) = F'(x)}$$

Example. The CDF of the random variable X has the form:

$$F(x) = \begin{cases} 0, & x < 0 \\ \frac{1}{33}(2x^2 + 5x), & 0 \le x \le 3 \\ 1, & x > 3 \end{cases}$$

Find PDF; E(X); var(X); $\sigma\ (X), P(1 < X < 2)$; plot graphs of CDF and PDF.

Example

Example. The PDF of the random variable X has the form:

$$f(x) = \begin{cases} \dfrac{C}{x^4}, & x \geq 2 \\ 0, & x < 2 \end{cases}$$

Find C; CDF; E(X); var(X); $\sigma\ (X)$; $P(2 < X < 3)$, plot graphs of CDF and PDF.

Tasks

Tasks

1. The random variable X is given on the entire *Ox* axis by the distribution function $F(x) = \frac{1}{2} + (arctgx)/\pi$. Find the probability that, as a result of the test, the random variable X will take on a value contained in the interval (0.1).

2. The random variable X (uptime of some device) is given on the entire axis by the distribution function $F(x) = 1 - e^{-\frac{x}{T}}\,(x \geq 0)$. Find the probability of failure-free operation of the device over time $x \geq T$.

3. The PDF of a continuous random variable X has the form $f(x) = \frac{2C}{1+x^2}$. Find the constant parameter C.

Tasks

4. The CDF of the random variable X has the form:

$$F(x)=\begin{cases}0,\ x\le 0\\ \sqrt{2}\sin x,\ 0<x\le\dfrac{\pi}{4}\\ 1,\ x>\dfrac{\pi}{4}\end{cases}$$

Find PDF; E(X); var(X); plot graphs of CDF and PDF.

5. The PDF of the random variable X has the form:

$$f(x)=\begin{cases}A(x-1), & x\in[2;4]\\ 0, & x\notin[2;4]\end{cases}$$

Find A; CDF; plot graphs of CDF and PDF; $P(3<X<7)$; E(X); var(X); $\sigma(X)$.

Tasks

6. The CDF of the random variable X has the form:

$$F(x) = \begin{cases} 0, & x < 0 \\ 1 - \cos x, & 0 \le x \le \dfrac{\pi}{2} \\ 1, & x > \dfrac{\pi}{2} \end{cases}.$$

Find PDF; E(X); var(X); plot graphs of CDF and PDF.

Homework

1. There are three tasks in the examination paper. The probability of a correct solution by a student of the first problem is 0.9, the second - 0.6, the third - 0.1. Find the distribution of the number of correctly solved problems and find its mathematical expectation, variance, standard deviation. Write down the CDF and plot its graph.

2. The distribution of the random variable X has the form:

X	-3	-2	1	3
p	1/7	3/7	2/7	1/7

Find its mathematical expectation, variance and standard deviation. Find the CDF of the random variable X and plot it.

3. The PDF of the random variable X has the form:

$$f(x) = \begin{cases} 0, & x < 1; \\ A\sqrt{x}, & 1 \le x \le 4; \\ 0, & x > 4 \end{cases}$$

Find A; $P(2 < X < 3)$; CDF; plot graphs of CDF and PDF; E(X); var(X); $\sigma(X)$.