Lecture 10

Function of one/two random arguments, system of two random variables

Function of one random argument

If each possible value of the random variable X corresponds to one possible value of the random variable Y, then Y is called a function of the random argument $Y = \varphi(x)$.

Example. The discrete random variable X is given by the distribution

X	3	6	10
p	0,2	0,1	0,7

Find the distribution of random variable $Y = 2X + 1$.

Function of two random arguments

If each pair of possible values of random variables X and Y corresponds to one possible value of random variable Z, then Z is called a function of two random arguments X and Y:

$$Z = \varphi(X, Y)$$

Example. Discrete independent random variables X and Y are given by distributions:

X	1	3
p	0,3	0,7

Y	2	4
p	0,6	0,4

Find the distribution of random variable $Z = X + Y$.

System of two random variables

A two-dimensional random variable $(\boldsymbol{X}, \boldsymbol{Y})$ is one whose possible values are pairs of numbers (x, y). The components X and Y, considered simultaneously, form a system of two random variables.

Covariance is a measure of the joint variability of two random variables. *If higher values of one variable generally correspond to higher values of the other variable, and the same is true for lower values, the covariance is positive*. Conversely, *when higher values of one variable generally correspond to lower values of the other, the covariance is negative*. **The sign of the covariance thus indicates the tendency of the linear relationship between the variables.**

$$cov(X, Y) = E(XY) - E(X)E(Y)$$

System of two random variables

Correlation is a measure of the relationship between random variables. This relationship is expressed through a correlation coefficient, which ranges from -1 to 1.

$$\rho(X,Y) = \frac{cov(X,Y)}{\sqrt{varX}\sqrt{varY}}$$

If X, Y are independent, then $cov(X,Y) = 0$.

System of two random variables

Example. A tetrahedral die is tossed twice. Let a and b be the scores on the first and second tosses.

Find the joint distribution of $X = \max\{a, b\}, Y = a + b$; $cov(X, Y)$; $\rho(X, Y)$.

Example. The joint distribution of random variables X and Y is given.

Find: -the distribution law of X, Y

-the covariance of X, Y

-the correlation of X, Y

-are X, Y independent?

$X \backslash Y$	-3	2	4	Sum
1	0.1	0.2	0.2	0.5
3	0.3	0.1	0.1	0.5
Sum	0.4	0.3	0.3	

Tasks

Tasks

1. The discrete random variable X is given by the distribution

X	-1	-2	1	2
p	0,3	0,1	0,2	0,4

Find the distribution of random variable $Y = X^2$.

2. Discrete independent random variables X and Y are given by distributions:

X	4	10
p	0,7	0,3

Y	1	7
p	0,8	0,2

Find the distribution of random variable $Z = X + Y$.

Tasks

3. A coin is tossed twice. Let X be the number of heads and Y be the number of tails. Find the joint distribution of X, Y and the marginal distributions.

4. The discrete random variable X is given by the distribution:

x	-2	-1	1	2
$f(x)$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

$Y = X^3$.

Find:

-distribution of Y

-joint distribution of X, Y

-covariance and correlation of X, Y

-are X, Y independent?

Tasks. Preparing for the test 2

5. Random variable X has a normal distribution with mathematical expectation $a = 5$ and variance $var(X) = 2$. Find the probability of this random variable falling on the interval (10;12).

6. Find the mathematical expectation and variance of a random variable X, distributed uniformly in the interval (1;5). Find and plot the graphs of the distribution function and distribution density of a uniform continuous random variable X.

7. It is known that out of 100 available guidebooks, there are 10 defective ones. 5 of them were randomly selected. Construct a distribution of the number of defective guidebooks contained in the sample. Find the mathematical expectation, dispersion, and standard deviation of this random variable.

Tasks. Preparing for the test 2

8. There are 100 tickets issued for the cash lottery. One win of 50 rubles and 10 wins of 1 ruble are played. Find the distribution of random variable X-cost of possible winnings.

9. Find the variance of a discrete random variable X—the number of occurrences of event A in five independent trials, if the probability of occurrence of events A in each trial is 0.2.

10. Three shots are fired at the target. The probability of hitting the target by the first shooter is 0.4, by the second – 0.5, by the third – 0.6. Random variable X – the number of hits on the target. Find distribution of the random variable X, E(X); var(X); plot graph of CDF.

Homework

1. The discrete random variable X is given by the distribution:

X	3	6	10
p	0,2	0,1	0,7

Find the distribution of random variable $Y = 3X - 5$

2. The discrete random variable X is given by the distribution:

X	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$
p	0,2	0,7	0,1

Find the distribution of random variable $Y = sinX$

Homework

3. The discrete random variable X is given by the distribution:

x	-2	-1	1	2
$f(x)$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

$Y = X^2$.

Find:

-distribution of Y

-joint distribution of X, Y

-covariance and correlation of X, Y

-are X, Y independent?