Lecture 11

Multicriteria optimization problems

Multicriteria optimization problems

To solve multicriteria optimization problems, the Pareto optimality criterion is used, the essence of which is to improve some indicators, provided that others do not deteriorate.

Features of multicriteria problems

1) The criteria are usually contradictory, that is, there is no better solution than others
2) The set of feasible alternatives is called the Pareto set
3) The northeastern boundary of the image of the Pareto set is the set of effective values

Bicriteria optimization problem

Consider a one-dimensional vector optimization problem:

$$f(x) = \big(f_1(x), f_2(x)\big) \to max$$
$$x = (x_1, x_2, \dots, x_n), x \epsilon R^n$$

The set X of feasible options (admissible solutions) is a region of n dimensional space R^n, called the space of alternatives.

Bicriteria optimization problem. Example (graphic solution)

Solve an optimization problem:

$$f_1(x) = 2x_1 - 3x_2 \rightarrow max$$
$$f_2(x) = x_1 + 3x_2 \rightarrow max$$

On set:

$$\begin{cases} x_1 - x_2 \leq 5 \\ x_1 + 2x_2 \leq 8 \\ -x_1 + 2x_2 \leq 4 \\ x \geq 0 \end{cases}$$

Reduction to a parametric linear programming problem

Consider a scalar problem with a parameter $\alpha \in [0; 1]$:

$$g(x, \alpha) = \alpha f_1(x) + (1 - \alpha) f_2(x) \to max$$

Point x^* is Pareto optimal if and only if there exists $\alpha \in (0; 1)$ such that x^* is a solution to the problem formulated above.

Reduction to a parametric linear programming problem

Solve the optimization problem:

$$f_1(x) = x_1 + 2x_2 \to max$$
$$f_2(x) = x_1 - x_2 \to max$$

On the set:

$$\begin{cases} 2x_1 + x_2 \leq 4 \\ x_1 - x_2 \leq 1 \\ x \geq 0 \end{cases}$$

Ideal point method

The ideal point method involves finding the point on the Pareto frontier that is closest to the utopia point.

Usually this point is not realized under given restrictions, which is why it is called the utopia point

Ideal point method. Example (graphic solution)

Find the Pareto-optimal solution, utopia point and ideal point:

$$f_1(x) = x_1 - x_2 + 5 \rightarrow max$$
$$f_2(x) = x_1 + x_2 - 2 \rightarrow max$$

On set:

$$\begin{cases} 2 \leq x_1 \leq 10 \\ 0 \leq x_2 \leq 6 \\ 3x_1 + 5x_2 \leq 45 \end{cases}$$

An example of solving an economic problem with two efficiency criteria

The flour mill sells premium baking flour in two ways: through a network of stores and through direct deliveries under contracts to non-trading organizations. It is known that stores can sell no more than 50 thousand monthly, and monthly supplies to non-trading organizations should not exceed 35 thousand tons of flour. No more than 45 thousand tons of flour are allocated for sale each month. The company developed a specific pricing policy that it intended to follow. However, due to the greatly changed economic situation, sales costs have increased, and flour has been included in the list of products that must be sold at a previously established price, regulated by local authorities. When selling one ton of flour through stores, sales costs began to amount to 7 thousand rubles, but the price remained the same - 10 thousand rubles; with the second sales method, the costs and price were 4 and 6 thousand rubles. respectively. It is necessary to determine how many tons of flour should be sold by each method so that costs are minimal and revenue from sales is maximized.