Scientific Computing Software. Maple

Lecture 1. Basics of Maple

Getting to know Maple. Document Mode and Worksheet Mode. Math Mode and Text Mode

Maple command syntax . Basic objects and data types.

Expression evaluation operations.

Expression transformation commands

Nasedkina A. A.

Getting to Know Maple

What is Maple?

User interface:

- Document Mode
- Worksheet Mode

Input modes and their features

- Text Mode
- Math Mode

Switching interface modes

Using Help

What is Maple?

- Computer Algebra Package, Environment for

exact and approximate calculations

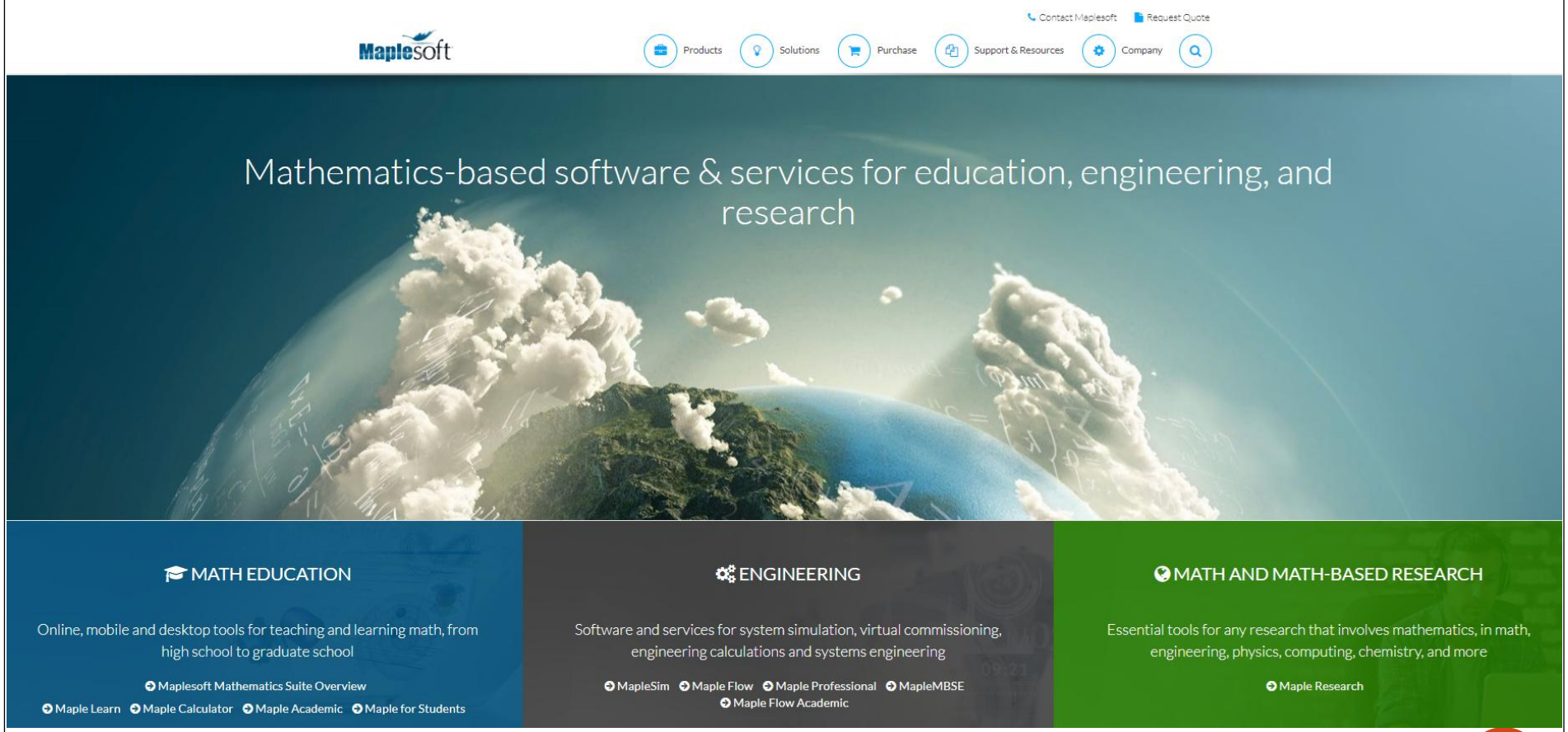
- Environment for **symbolic (analytical) calculations** , with tools for approximate calculations (numerical solution of algebraic and differential equations, calculation of derivative and integrals, solving problems of linear algebra, statistics, etc.)
- Advanced graphics, visualization
- A programming language resembling Pascal

Developer: Waterloo Maple Inc. (concept: University of Waterloo, Waterloo, Ontario, Canada, 1980)

Website: maplesoft.com

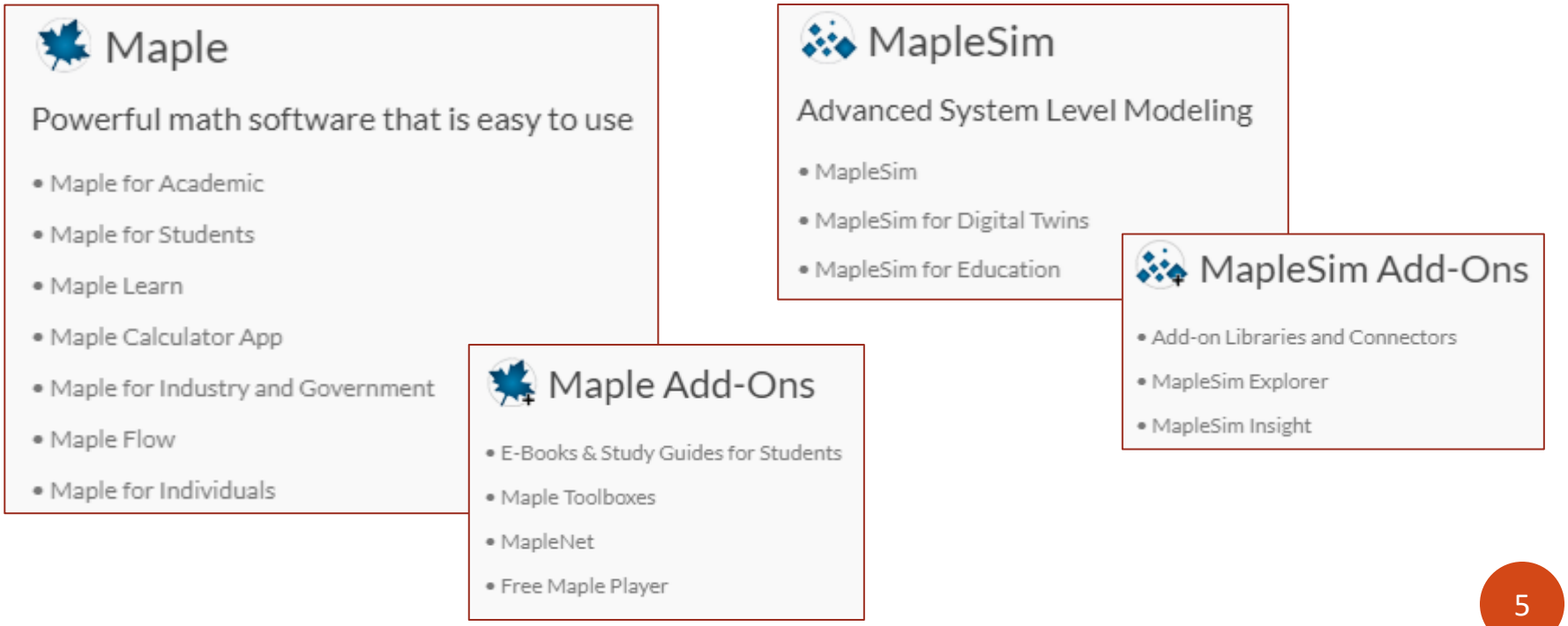
Maple today

- www.maplesoft.com



Maple

- First release: Maple 1.0 (1982)
- Next releases: Maple 2-18 (1982-2014) , then Maple 2015 and further version names by year
- Latest version: Maple 2026
- There are professional, academic, student and other versions (all paid, commercial software)



Version of Maple in our Institute

- **Our license: Maple 11.0 (2007)**

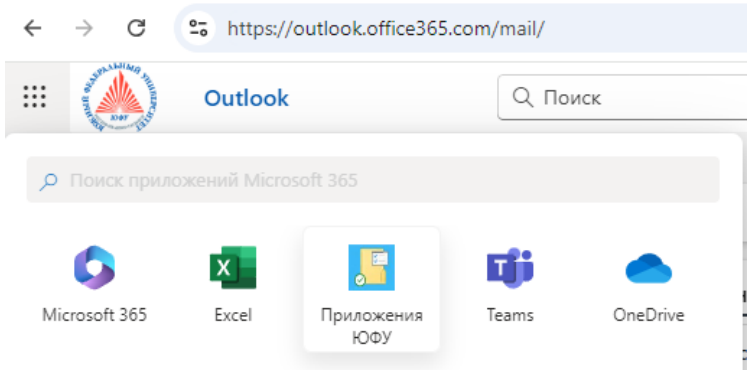
Maple 11.0 Features

- You can enter mathematical expressions using standard notations
- There are interfaces to C, C#, Fortran, Java, Matlab , VisualBasic , and Excel.
- The functionality is provided by libraries

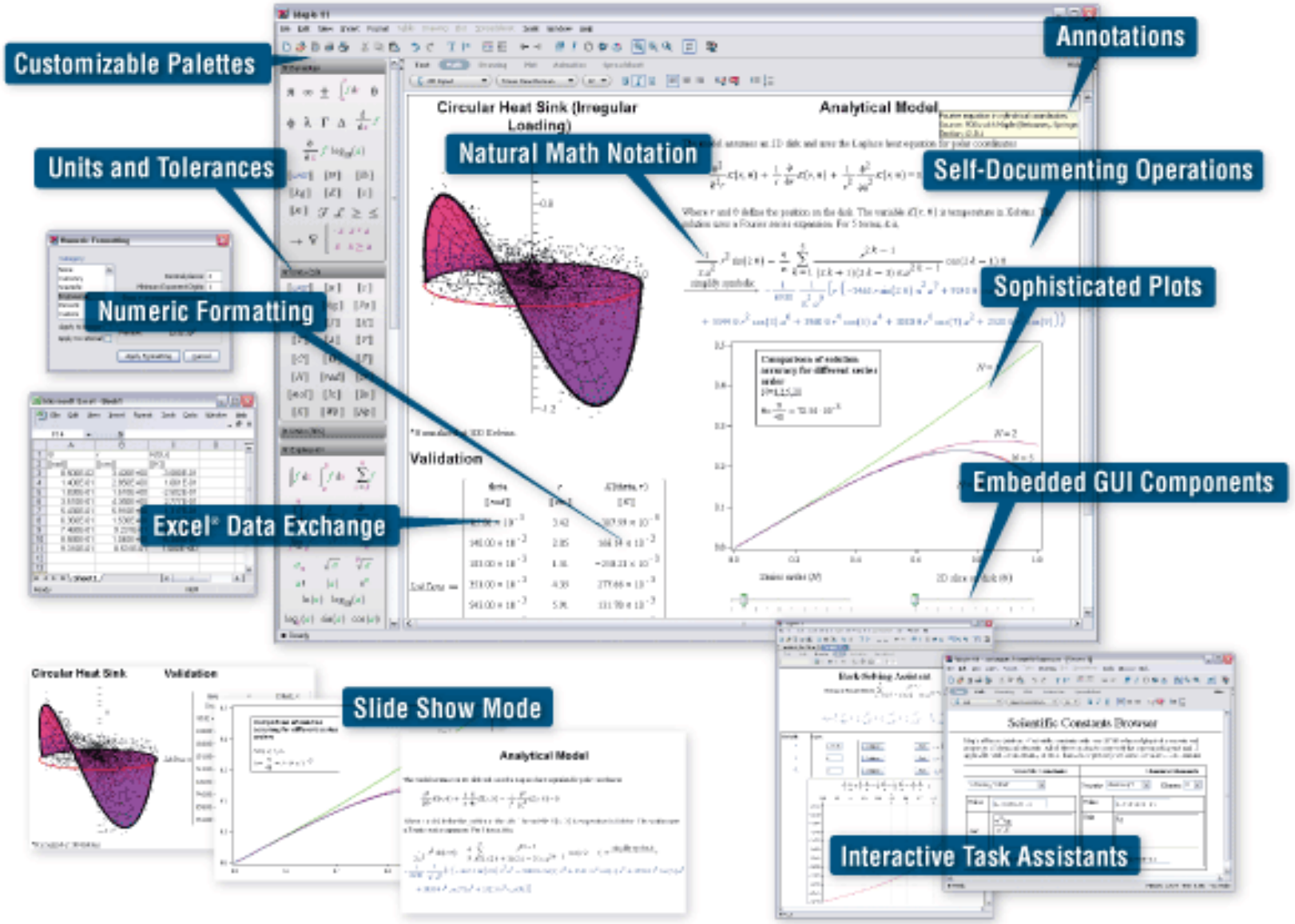
Working with Maple 11

- Maple 11 is installed in many computer classes in our Instigute (LOS-1-2, a. 101, 102, 118, 202, etc.)
- Maple 11 can be accessed through SFedU Remote Applications https://rdf01.sfedu.ru/RDWeb/Pages/en-US/login.aspx

@sfedu.ru email inbox in Outlook , clicking on the dotted icon (application launcher) in the upper left corner and selecting SFedU Applications, entering your SFedU account login and password.



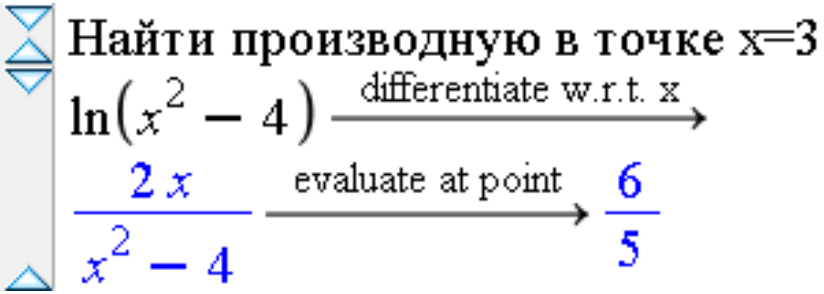
Key Features and Capabilities of Maple 11



Interface modes in Maple

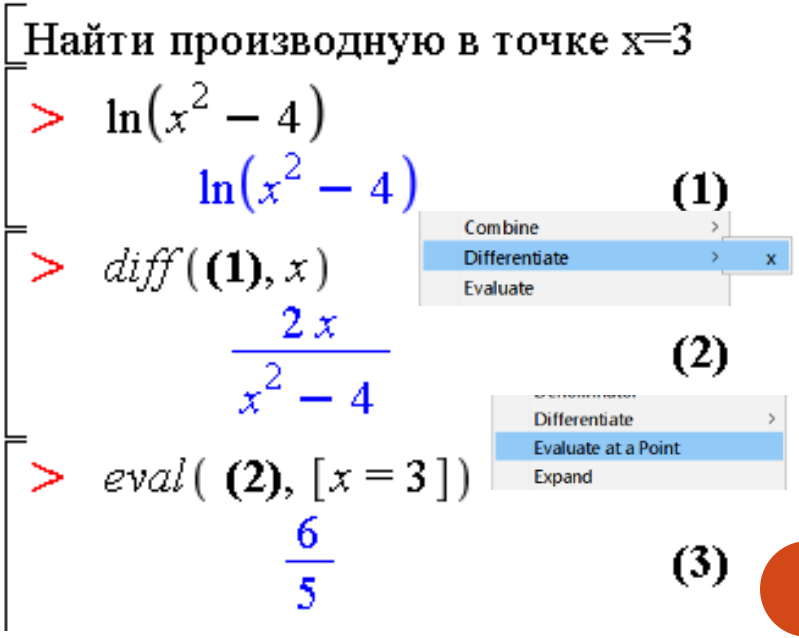
Document Mode

- A mode for creating documents with mathematical calculations
- Hides all commands used to perform calculations.
- Knowledge of Maple syntax is not required.



Worksheet Mode

- Maple's traditional command-line interface
- There is a command line symbol: execution group [>
- All command syntax is displayed.



Entering mathematical expressions

- Input modes: **Text Mode** and **Math Mode** (switch – [F5])
- **Math mode** : for entering mathematical expressions in the style of "two-dimensional mathematics" 2D Math: $\frac{1}{2}+\frac{3}{5}, x^2, a_1$
- **Text Mode** : in the Document Mode interface – for entering text comments , in the Worksheet Mode interface – for entering mathematical expressions in the 1D Math style: `1/2+3/5, x^2, a_1`

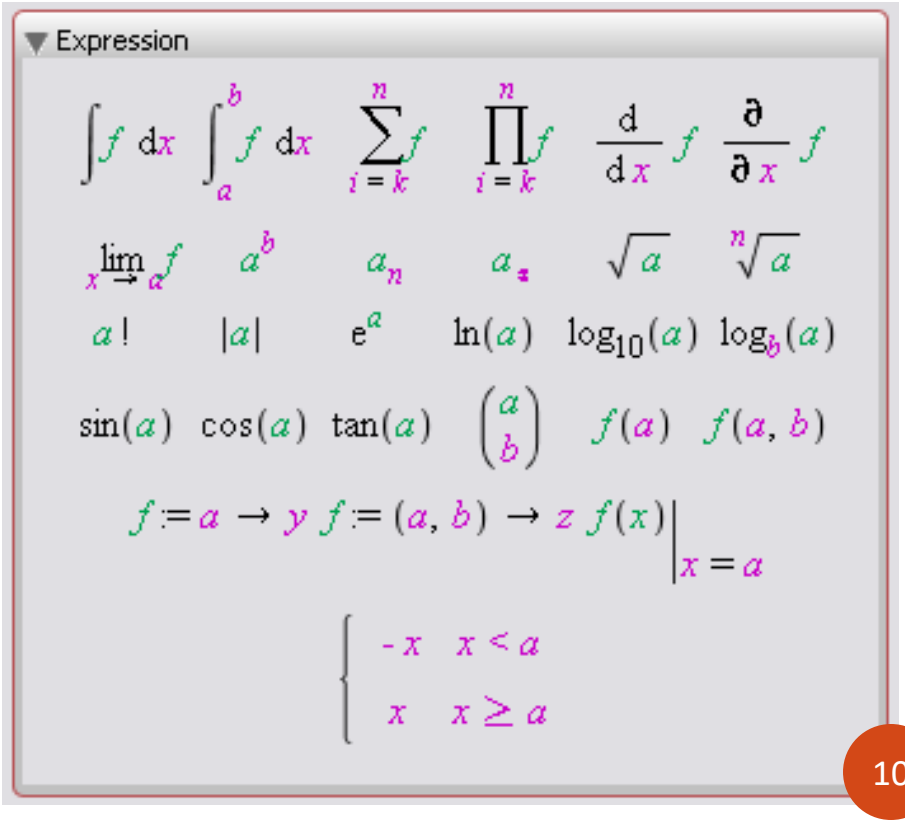
2D-Math

- Navigation by expression - using buttons
- Use buttons to navigate through entering rational fractions or powers [←][→][↑][↓]
- The multiplication sign * can be omitted in cases multiplying a number by a variable $2x$ or a variable by a variable (a space is placed between the variable names): $x\,y$

Using Pallets to Enter Complex Mathematical Expressions

$$\sin\left(\frac{\pi}{3}\right) + i\cos\left(\frac{\pi}{4}\right)$$

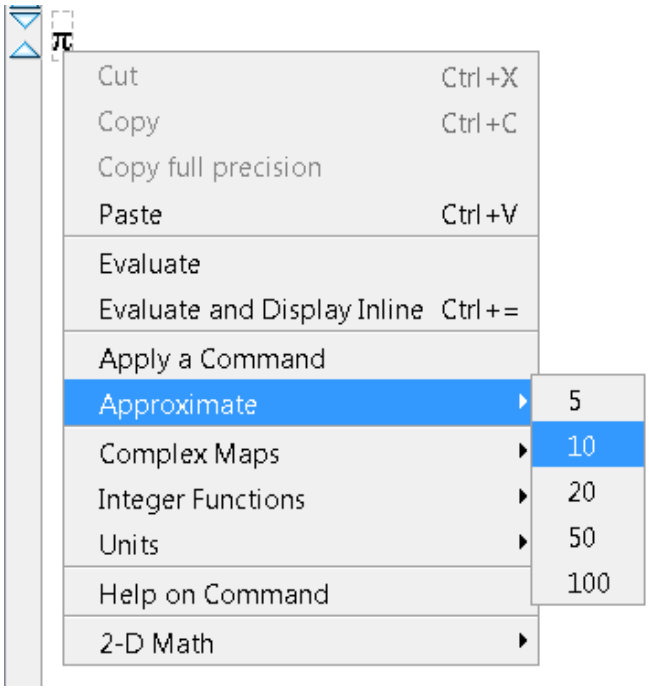
$$\int_0^{1.9} x^2 \sin(x^2+1)\, dx$$



Additional features for entering and calculating mathematical expressions

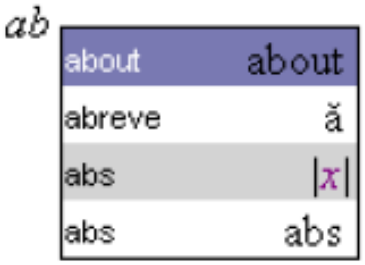
Calling the context menu

- Right mouse button



Automatic command completion

- **[Ctrl] + [Space]**



Execution buttons

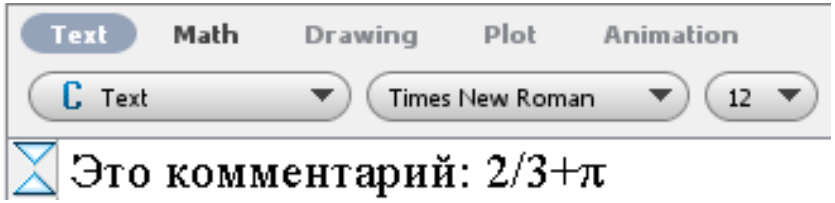
- Execute calculations for the selected rows or the current row: 
- Execute calculations for the entire document: 

Document Mode

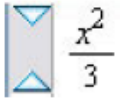
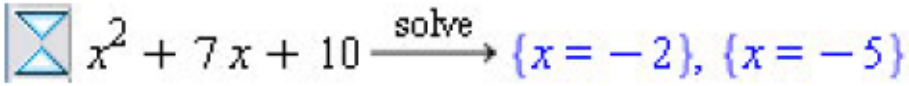
- File->New->Document Mode
- Enabling document block markers View->Mar  s

Text Mode input mode

- Vertical cursor in a document block 
- For plain text input, mathematical expressions are not calculated.



Math Mode input mode

- Italic cursor in a document block 
- Input in 2D-Math form 
- Using the context menu, all commands are hidden, only the result is shown 

Document Mode interface mode : expression evaluation

Display result in the same line

- **[Ctrl] + [=]**
- Context menu: Evaluate and Display Inline

$$\sin\left(\frac{\pi}{3}\right) + i\cos\left(\frac{\pi}{4}\right) = \frac{1}{2}\sqrt{3} + \frac{1}{2}i\sqrt{2}$$

Displays result on the next line and centered

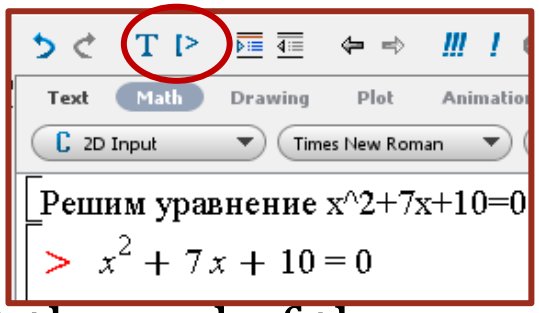
- [Enter]
- Context menu: Evaluate

$$\int_0^{\pi} \sin(x)\, dx$$

$$2$$

Worksheet Mode

- File->New->Worksheet Mode
- Both text input modes – Math and Text – are used to enter mathematical expressions and Maple commands.
  - **Math** – input in 2D-Math notation $\frac{1}{2}+\frac{3}{5}, \mathrm{sqrt}(2)$
  - **Text** – input in 1D-Math notation `1/2+3/5, x^2, a_1`
- Entering text comments : T
- Insert workgroup : [>
- Separation of teams from each other – **;** or **:**
- Suppressing output to the screen - a colon at the end of the command



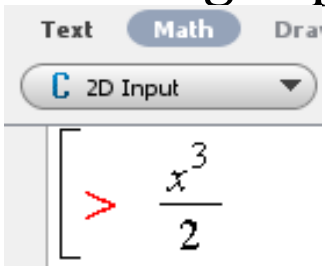

```
> 2 + 3 :
> 2 + 3 : sqrt(2); evalf(π, 3)
```

$\sqrt{2}$

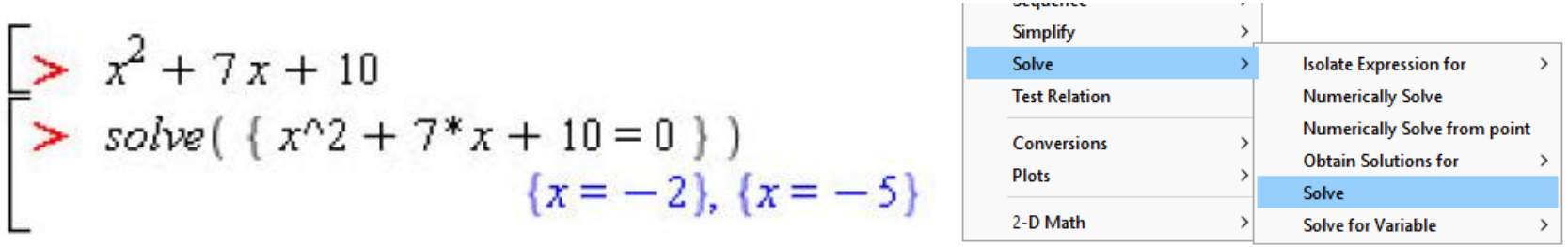
3.14

Math Mode input mode in Worksheet Mode

- Tilted cursor in execution group [> [
- Entering expressions in 2D-Math



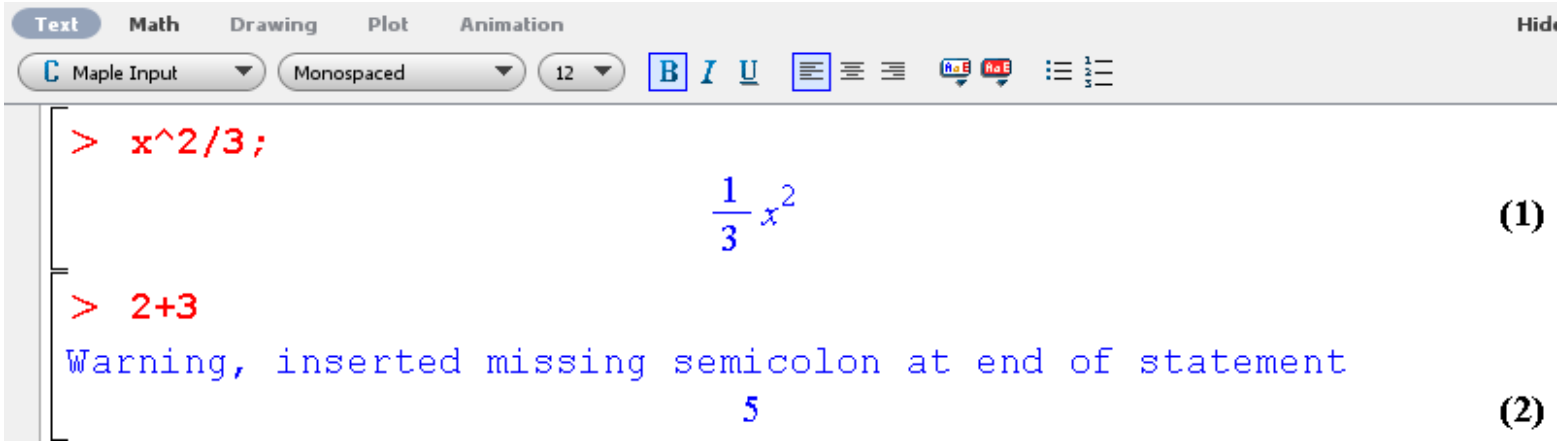
- Executing commands – by pressing [Enter]
- All command syntax is displayed when using the context menu.



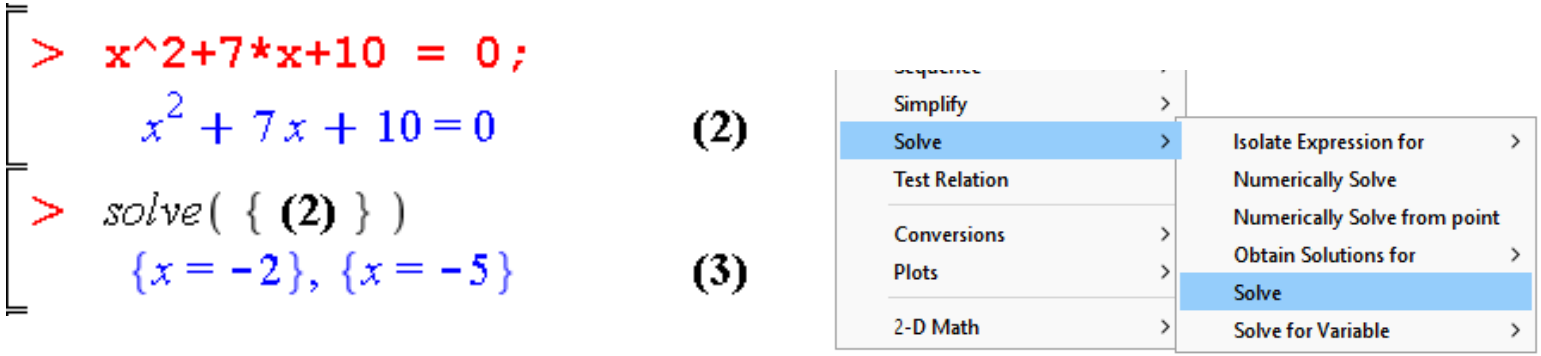
- The context menu can be called both for the entered expression and for the result.

Text Mode in Worksheet Mode

- Old Maple style
- Vertical cursor in the execution group [>
- Entering expressions in the form 1 D-Math , the entered expression must end with a semicolon or colon



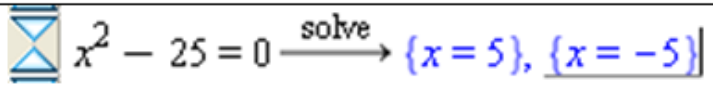
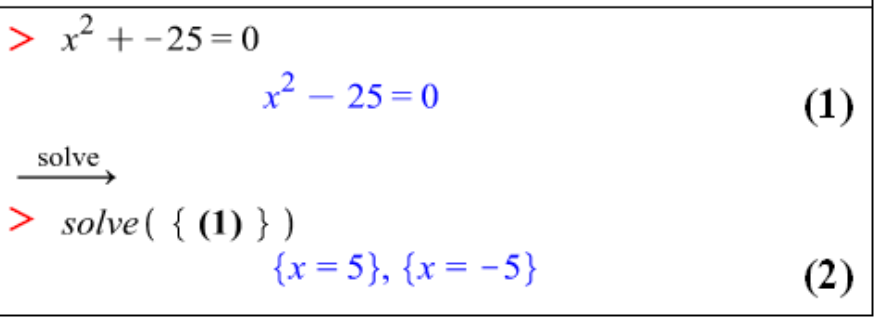
- The context menu can only be called for the result!



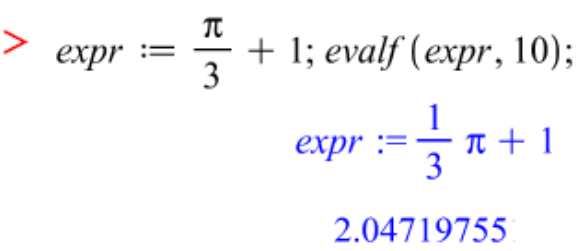
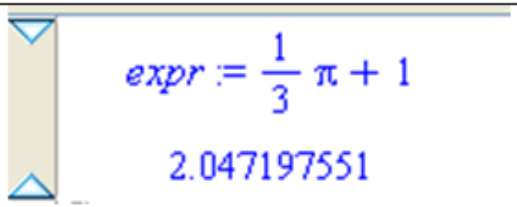
Switching between interface modes

- In Document Mode, you can show hidden commands:

View -> Expand Document Block :

Document Mode	Worksheet Mode
	

- In Worksheet Mode, you can hide all commands and show only the result: View -> Collapse Execution Group

Worksheet Mode	Document Mode
	

Combining different interface modes

- From Document Mode to Worksheet Mode

1. Inserting an execution group using a button [>
2. Inserting an execution group before or after the cursor, using the correct menu: Insert -> Execution Group -> Before / After Cursor

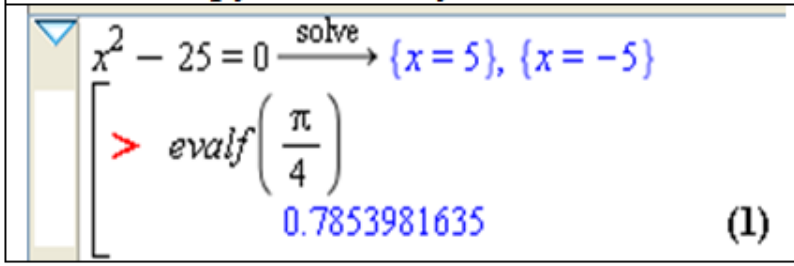
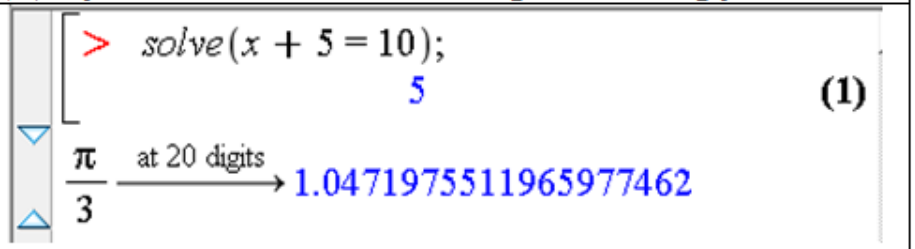
- From Worksheet Mode to Document Mode

Format -> Create Document Block

Format -> Remove Document Block

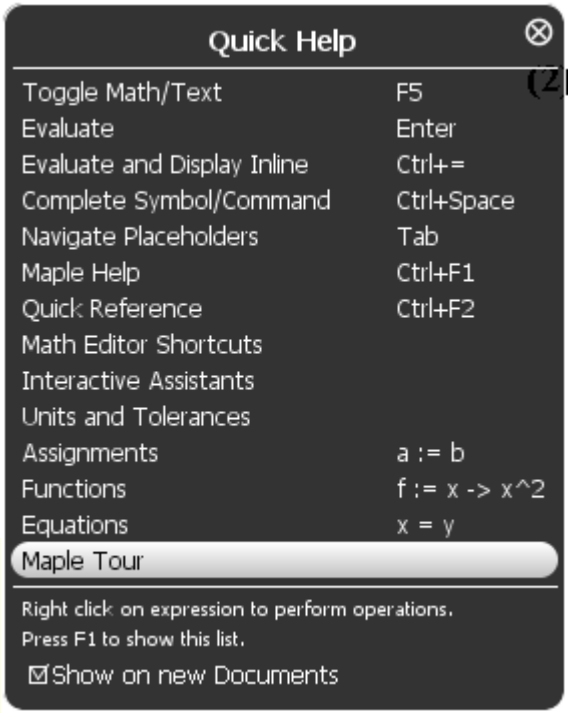
- Deleting a workgroup element or document block

Edit -> Delete Element or the keyboard shortcut [Ctrl]+[Del]

Режим интерфейса Document Mode Рабочая группа в документном блоке	Режим интерфейса Worksheet Mode Документный блок после рабочей группы
	

Using the Maple Help System

- [F1] – Quick Help
- [F2] – command help
- [Ctrl]+[F1] – opens the help system
- [Ctrl]+[F 2] – Quick Reference



Maple Quick Reference Card

Windows® version

Document Mode vs. Worksheet Mode

Common Operations Available in Both Document and Worksheet Modes

Display quick help	**[F1]** for Quick Help. **[Ctrl] [F2]** for Quick Reference Card (this guide)
Refer to previous result using equation numbers	**[Ctrl] [L]** then enter equation number in dialog
Recompute calculations within a highlighted selection or chain of commands	! on toolbar
Recompute all calculations in a document	!!! on toolbar
Symbol selection, e.g. ϵ (epsilon)	Enter leading characters **[Ctrl] [Space]**, e.g. **eps [Ctrl] [Space]**
Command completion, e.g. Lambert W function	Enter leading characters **[Ctrl] [Space]**, e.g. **Lamb [Ctrl] [Space]**
Perform context operation on math expression	Right-click any math expression
Insert prompt	[> on toolbar
Insert text paragraph	T on toolbar

2-D Math Editing Operations, Keyboard Shortcuts, and Operations (Details)

Maple

Command syntax

Using command packages

restart command

Operators :=, #, %

Numbering and links to results

Maple

- Top-level commands are available directly
- **command(arg1,arg2,…);**

```
> diff( tan(x) sin(x) , x)
```

$$\left(1+\tan(x)^2\right)\sin(x)+\tan(x)\cos(x)$$

- Commands from packages require the use of specific package:
- **package[command](arg1,arg2,…);**
- **with(package): command(arg1,arg2,…);**

Команда создания единичной матрицы из пакета LinearAlgebra

```
> LinearAlgebra[IdentityMatrix](2);
```

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

```
> with(LinearAlgebra) :
> IdentityMatrix(2)
```

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Maple Syntax : Using Command Packages

- Enabling the use of a package

➢ **with(package):**

command(arg1,arg2,...);

```
> with(Optimization):
```

```
> NLPSolve(sin(x)/x, x=1..15)
```

$$[-0.09132520282305767 18, [x = 10.904121670074490 0]]$$

- Disabling a package

unwith (package):

```
> unwith(Optimization):
```

- An example where disabling a package is required: there are two identical **changecoords** commands – top-level and in the plots package

Important operators

- Assignment operator **:=**
- **restart** command clears memory.
- Comment mark **#** (ignoring input to the end of the line)
- Calling the previous result **%**
  - pre-previous - and pre-previous result **% %** And **%%%**

```
> a := 2
```

$$a := 2$$

```
> a + 3
```

$$5$$

```
> restart; a
```

$$a$$

```
> b := 1 : #c:=2
> a := b + c
```

$$a := 1 + c$$

```
> % - 3
```

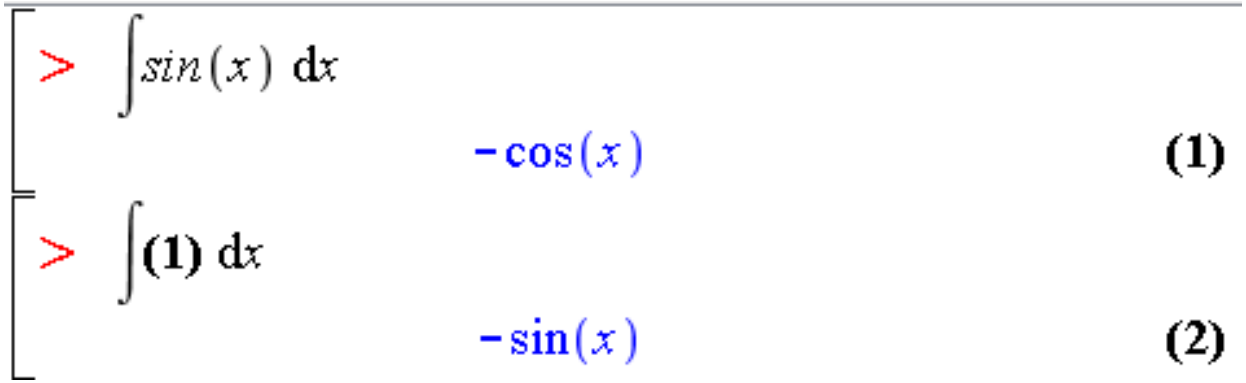
$$-2 + c$$

```
> %%% - %
```

$$3 - c$$

Numbering and labels to results

- All calculation results are numbered (both in Worksheet Mode and Document Mode) and can be referenced.



- Insert->Label or [Ctrl]+[L]

- If the numbering is not displayed:

Tools->Options->Display->Show equation labels

Basic objects, types of calculations and data types in Maple

Maple's

- Numbers
- Constants
- Symbolic objects (names)
- Variables

Types of calculations

Basic data types

Main objects in Maple

- The simplest objects in Maple are:
  - Numbers
  - Constants
  - Symbolic objects (names)
  - Variables
-
- Simple Maple objects can be used to form *expressions* Maple , using signs of mathematical operations (mathematical expressions), logical operations (logical expressions), etc.

Numbers

- **Целые (*integers*)**
  Пример: 3, −100
- **Рациональные (обыкновенные дроби)**
  Maple 2D-Math: > $\frac{2}{3}$
  Maple 1D-Math: > `2/3;`
- **Иррациональные (радикалы)**
  Maple 2D-Math: > $\sqrt{2}$; $\sqrt[3]{123}$
  Maple 1D-Math: > `sqrt(2); 123^(1/3);`
- **Комплексные**
  Maple 2D-Math: > $2 + 3 \cdot I$
  Maple 1D-Math: > `2+3*I;`
- **Числа с плавающей точкой (*floating-point*)**
  Задаются в виде целой и дробной частей, разделенных десятичной точкой, либо в показательной (экспоненциальной) форме
  Maple 2D-Math: > 1.02
  Maple 1D-Math: > `0.102e1;`

Constants

- Maple contains several predefined named (symbolic) constants.
- The value of constants can be accessed using their names or the symbols that correspond to them.
- A constant in Maple is an object whose value cannot be changed using the assignment operator.
  - Mathematical constants π; I; ∞
  - Logical constants **true** , **false** , **FAIL**
  - Scientific constants (available with the Scientific Constants package), categories: physical constants and constants of the properties of chemical elements and isotopes

```
> with(ScientificConstants)
[AddConstant, AddElement, AddProperty, Constant, Element, GetConstant, GetConstants, GetElement, GetElements,
  GetError, GetIsotopes, GetProperties, GetProperty, GetUnit, GetValue, HasConstant, HasElement, HasProperty,
  ModifyConstant, ModifyElement]
> GetConstant(c)
```

$$speed\_of\_light\_in\_vacuum,\ symbol = c,\ value = 299792458,\ uncertainty = 0,\ units = \frac{m}{s}$$

Symbolic objects (names)

- A symbolic object is any combination of letters, numbers, and underscores beginning with the letter , without the use of mathematical operators. Names may contain letters of the national alphabet, including Russian.

```
> a:
> b12_c3:
> ElenaIvanova:
> 12v;
```

- `Error, missing operator or ;` any characters enclosed in backticks .

```
> `12v`;
```

$$12v$$

```
> `Sasha+Masha`;
```

$$Sasha + Masha$$

Variables

- Symbolic objects are used to declare *variables* . The value of a variable is set using the assignment operator:

variable name := variable value

- Variable names can be any user-defined allowed name or any combination of characters enclosed in backticks.
- You cannot use reserved names, such as Maple constant names , Maple command names , system variables, logical operators, programming structure words, etc. (even when enclosed in backticks).

Examples of defining variables

```
> a:=3;
```

$$a := 3$$

```
> `12v`:=10;
```

$$12v := 10$$

```
> a:=`12v`+1;
```

$$a := 11$$

```
> Pi:=1;
Error, attempting to assign to `Pi` which is protected
> plot:=a+1;
Error, attempting to assign to `plot` which is protected
> `plot`:=a+1;
Error, attempting to assign to `plot` which is protected
```

Cancellation of assignment

- variable name **:=** ' variable value '
- **unassign (' a ')**

```
> b:=2;c:=8;b+1;c+5;
```

$$b := 2$$

$$c := 8$$

$$3$$

$$13$$

```
> b:='b';b+1;
```

$$b := b$$

$$b + 1$$

```
> unassign('c');c+5;
```

$$c + 5$$

Types of calculations in Maple

- **symbolic (exact) :** analytical transformations of expressions containing symbolic quantities, variables, functions and exact numbers.

```
> sqrt(2)/(3+5);
```

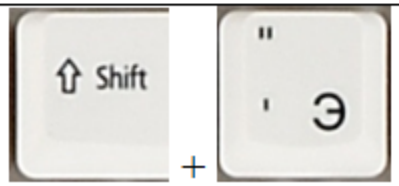
$$\frac{1}{8}\sqrt{2}$$

- **numerical (approximate, floating - point computations) :** finite-precision arithmetic is used; such calculations use approximate values of exact quantities; such calculations always contain some error.

```
> sqrt(2.)/(3+5);
```

0.1767766952

Quotation marks used in Maple

Вид кавычек	Обозначение	Набор на клавиатуре EN-раскладка	Значение
Обратные одинарные кавычки	`` `...` ``		Задают символьный объект
Прямые одинарные кавычки	`'...'`		Используются в синтаксисе отложенных вычислений
Прямые двойные кавычки	`"..."`		Задают строку

Basic data types

In Maple, there are approximately 200 data types, including :

- types of mathematical operations:
  - `+` (addition)
  - * (multiplication)
  - `^` (raising to a power)
- comparison types:
  - `<` (less)
  - `<=` (less than or equal to)
  - `<>` (not equal to)
- types of logical operations
- numeric types, for example:
  - **integer** – whole number
  - **float** – a floating-point number
  - **fraction** – rational fraction
- symbol type

Data type definition commands

- **whattype (x)** – returns the type of object *x*
- **type(x,x_type)** – checks whether an object *x* matches the type x_type and returns one of the logical constants: **true** , **false** , or **FAIL** (undefined)

```
> whattype(a+b);whattype(a-b);
                                    `+`
                                    `+`
> whattype(2>3);
                                    `<`
> whattype(1/2);
                                  fraction
> whattype(2);
                                  integer

> whattype(2.);
                                  float
```

Data Type Definition Commands: Examples

```
> whattype(a);whattype(`a+b`);
```

symbol

symbol

```
> type(2,integer);type(2,float);
```

true

false

```
> type(a+b,`+`);type(`a+b`,`+`);
```

true

false

- The type of a variable can change depending on the value assigned to it:

```
> a:=3; whattype(a);
```

$a := 3$

integer

```
> a:=sqrt(2);whattype(a);
```

$a := \sqrt{2}$

`^`

Expression evaluation operations

- eval
  - eval
  - evalf
  - evalb
  - evalc
  - evalm
- Evaluation of real expressions
- Evaluation of complex expressions

eval

- **eval (*x*)** – evaluates the value of the expression x
- **eval (expr , x = a)** – calculating the value of the expression expr when x=a

- **evalf (*x*)** – calculates the approximate value of the expression x

evalf [n](x) – approximate value of x with output of n digits of the number

- Environment variable **Digits** – specifies the number of digits in floating-point numbers.

Digits := n

(n is a natural number, by default n=10)

- **evalc** (*x*) – calculates the value of a complex expression
- **evalb** (*x*) – calculating the value of a logical expression
- **evalm** (*x*) – calculates the value of a matrix expression

eval and **evalf** commands: examples of use

Команды eval и evalf

> $f := x^3 + 5x + 2$

$$f := x^3 + 5x + 2$$

> $eval(f, x = \sqrt{2})$

$$7\sqrt{2} + 2$$

> $f\Big|_{x=3.2}$

$$50.768$$

> $f1 := eval(f, x = \sqrt{2}) : evalf(f1)$

$$11.89949493$$

> $evalf[15](f1)$

$$11.8994949366117$$

Использование переменной среды Digits

> $Digits := 20 : evalf(f1)$

$$11.899494936611665342$$

Other commands in the **eval** family: examples

Значение комплексного выражения

```
> z := -1 - I*sqrt(3); z^4
```

$$z := -1 - I\sqrt{3}$$

$$\left(-1 - I\sqrt{3}\right)^4$$

```
> evalc(z^4)
```

$$-8 - 8\,I\sqrt{3}$$

Значение булева выражения

```
> f := 1 = 2; eval(f); evalb(f);
```

$$f := 1 = 2$$

$$1 = 2$$

$$false$$

Значение матричного выражения

```
> A := Matrix([[1, 2], [3, 4]]): B := Matrix([[5, 6], [7, 8]]):
```

```
> evalm((A + B)^2)
```

$$\begin{bmatrix} 116 & 144 \\ 180 & 224 \end{bmatrix}$$

Evaluation of real expressions

trunc (*x*) – calculation of the integer part of the real expression x
frac (*x*) – calculation of the fractional part real expression x
round (*x*) – rounding of a real expression x

```
> trunc(8/3); frac(8/3); round(8/3)
```

$$2$$

$$\frac{2}{3}$$

$$3$$

```
> trunc(-2.5); frac(-2.5); round(-2.5)
```

$$-2$$

$$-0.5$$

$$-3$$

```
> trunc(3.5 + 4.2 I); frac(3.5 + 4.2 I); round(3.5 + 4.2 I)
```

$$3 + 4\,I$$

$$0.5 + 0.2\,I$$

$$4 + 4\,I$$

Evaluation of complex expressions

Re (z) – real part
Im (z) – imaginary part
conjugate (z) – complex conjugate of an expression
polar (z) – modulus and argument
evalc (z) – calculates the value of a complex expression

```
> z := 5 - 3*I
```

$$z := 5 - 3\,\mathrm{I}$$

```
> Re(z)
```

$$5$$

```
> Im(z)
```

$$-3$$

```
> conjugate(z)
```

$$5 + 3\,\mathrm{I}$$

```
> polar(z)
```

$$\mathrm{polar}\left(\sqrt{34},\ -\arctan\left(\frac{3}{5}\right)\right)$$

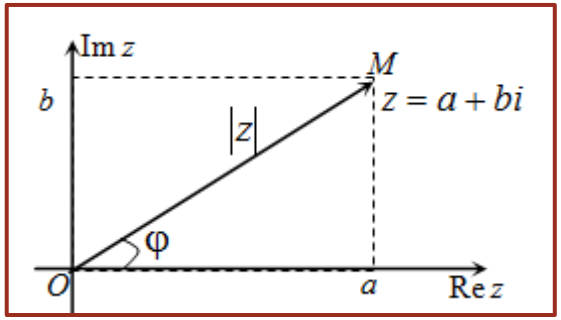
```
> z := exp(I*Pi/3)^2
```

$$z := \left(e^{I\cdot\frac{\pi}{3}}\right)^2$$

$$z := \left(\frac{1}{2} + \frac{1}{2}\,\mathrm{I}\sqrt{3}\right)^2$$

```
> evalc(z)
```

$$-\frac{1}{2} + \frac{1}{2}\,\mathrm{I}\sqrt{3}$$



Expression transformation commands

- Standard mathematical functions
- Commands for working with integers
- Selecting the right and left sides of an expression, the numerator and denominator of a fraction
- Converting a fraction to normal form
- Factorization of a polynomial
- Collecting similar terms in expression
- Expanding parentheses
- Transformation and simplification of expressions

Standard mathematical functions

Mathematical notation	1D-Math Input	2D-Math (pallets)
e^x	`exp(x)`	e^x
$\ln x$	`ln(x)`	$\ln(x)$
$\log_{10} x$	`log10(x)`	$\log_{10}(x)$
$\log_a x$	`log[a](x)`	$\log_a(x)$
$\lvert x \rvert$	`abs(x)`	$\lvert x \rvert$
$\sqrt{x}$	`sqrt(x)`	$\sqrt{x}$
$\operatorname{sgn} x$	`signum(x)`	$\operatorname{signum}(x)$
$\sqrt[n]{x}$	`root(x,n)`	$\sqrt[n]{x}$
$n!$	`n!`	$n!$
$\delta(x)$	`Dirac(x)`	$\operatorname{Dirac}(x)$

- In 2D-Math mode , most of these mathematical functions can be typed using templates in the Expression palette.
- In 2D-Math mode , these functions are displayed in upright font and their argument in italics!

Trigonometric and hyperbolic functions

Mathematical recording	Syntax Maple (1D-Math Input)	2D-Math
$\sin x$	`sin( x )`	$\sin(x)$
$\cos x$	`cos (x)`	$\cos(x)$
$\mathrm{tg}x$	`tan(x)`	$\tan(x)$
$\mathrm{ctg}x$	`cot(x)`	$\cot(x)$
$\mathrm{sh}x$	`sinh (x)`	$\sinh(x)$
$\mathrm{ch}x$	`cosh (x)`	$\cosh(x)$
$\mathrm{th}x$	`tanh (x)`	$\tanh(x)$
$\mathrm{ctg}x$	`coth (x)`	$\coth(x)$
$\sec x$	`sec(x)`	$\sec(x)$
$\mathrm{cosec}x$	`csc (x)`	$\csc(x)$

In 2D-Math mode , the names of trigonometric functions are always displayed in upright font and their argument in italics!

tg(x) #Здесь ошибка!

Inverse trigonometric and hyperbolic functions

Mathematical recording	Syntax Maple (1D-Math Input)	2D-Math
$\arcsin x$	`arcsin ( x )`	$\arcsin(x)$
$\arccos x$	`arc cos (x)`	$\arccos(x)$
$\mathrm{arctg}x$	`arc tan (x)`	$\arctan(x)$
$\mathrm{arcctg}x$	`arc cot (x)`	$\operatorname{arccot}(x)$
$\mathrm{arcsh}x$	`arc sinh (x)`	$\operatorname{arcsinh}(x)$
$\mathrm{arcch}x$	`arc cosh (x)`	$\operatorname{arcsinh}(x)$
$\mathrm{arcth}x$	`arc tanh (x)`	$\operatorname{arctanh}(x)$
$\mathrm{arcctg}x$	`arc coth (x)`	$\operatorname{arccoth}(x)$
$\mathrm{arcsec}x$	`arc sec (x)`	$\operatorname{arcsec}(x)$
$\mathrm{arccosec}x$	`arc csc (x)`	$\operatorname{arccsc}(x)$

In 2D-Math mode , the names of trigonometric functions are always displayed in upright font and their argument in italics!

Commands for working with integers

isprime (*n*) – a Boolean function that checks whether a number is prime
ifactor (*n*) – factorization into prime factors
irem (*n* , *m*) – calculation of the remainder after dividing n by m
iquo (*n* , *m*) – calculation of the quotient of n divided by m

```
> isprime(57)
                              false
> ifactor(57)
                           (3) (19)
> isprime(157)
                              true
> ifactor(157)
                            (157)
> irem(62, 4)
                               2
> iquo(62, 4)
                               15
```

Commands for working with integers

igcd (*n1 , n2, …*) – calculation of the greatest common divisor numbers n1 , n2, …
ilcm (*n1 , n2, …*) – calculation of the least common multiple numbers n1 , n2, …
isqrt (*n*) – integer square root
iroot (*n*) – integer root of the n-th degree

```
> igcd(-10, 6, -8)
```

$$2$$

```
> ilcm(-10, 6, -8)
```

$$120$$

```
> isqrt(8); sqrt(8); evalf(%)
```

$$3$$

$$2\sqrt{2}$$

$$2.828427124$$

```
> iroot(100, 3); evalf(root(100, 3))
```

$$5$$

$$4.641588834$$

Root extraction commands

root (*x* , *n*) – the n -th root of an algebraic expression (n is an integer)
surd (*x* , *n*) – the n -th root of x whose (generally complex) argument is closest to root of x
If n is odd and
x>=0 , then surd(x,n) = x^(1/n)
x<0 , then surd(x,n) = -(-x)^(1/n).

```
> (8)^(1/3); root(8, 3); surd(8, 3);
```

$$8^{1/3}$$
$$2$$
$$2$$

```
> (8.0)^(1/3); root(8.0, 3); surd(8.0, 3);
```

2.000000000
2.000000000
2.000000000

```
> (-8)^(1/3); root(-8, 3); surd(-8, 3);
```

$$(-8)^{1/3}$$
$$2\,(-1)^{1/3}$$
$$-2$$

```
> (-8.0)^(1/3); root(-8.0, 3); surd(-8.0, 3);
```

1.000000000 + 1.732050807 I
1.000000000 + 1.732050807 I
−2.000000000

Selecting the right and left parts of an expression : rhs , lhs

lhs (*expr*) – left-hand side selection
rhs (*expr*) – right-hand side selection

```
> eq := a^2 - b^2 = c
```

$$eq := a^2 - b^2 = c$$

```
> lhs(eq)
```

$$a^2 - b^2$$

```
> rhs(eq)
```

$$c$$

```
> h := x + y < 10
```

$$h := x + y < 10$$

```
> lhs(h)
```

$$x + y$$

```
> rhs(h)
```

$$10$$

Reducing a fraction (algebraic expression) to normal form : normal

normal (*x*) – converting a fraction (expression) to normal form
normal (*x* , *expanded*) – converting a fraction (expression) to normal form with bracket expansion in polynomials

> $normal\left(x^2-(x+1)\,(x-1)-1\right)$

$$0$$

> $normal\left(\frac{a^2-2\cdot a\cdot b+b^2}{(a-b)^2}\right)$

$$1$$

> $normal\left(\frac{x^2-y^2}{(x-y)^3}\right)$

$$\frac{x+y}{(x-y)^2}$$

> $normal\left(\frac{1}{x}+\frac{x}{x+1}\right); normal\left(\frac{1}{x}+\frac{x}{x+1}, expanded\right)$

$$\frac{x+1+x^2}{x\,(x+1)}$$

$$\frac{x+1+x^2}{x^2+x}$$

Identifying the numerator and denominator of a rational fraction : numer , denom

numer (*x*) – selection of the numerator of a fraction (algebraic expression)
denom (*x*) – selection of the denominator of a fraction (algebraic expression)

$$> \quad z := \frac{x^2 - y^2}{(x-y)^3} : z1 := normal(z)$$

$$z1 := \frac{x+y}{(x-y)^2}$$

$$> \quad numer(z); numer(z1)$$

$$x^2 - y^2$$

$$x + y$$

$$> \quad denom(z); denom(z1)$$

$$(x-y)^3$$

$$(x-y)^2$$

Factorization of a polynomial : factor

factor (p) – factorization of a polynomial

> $factor\left(6x^2 + 18x - 24\right)$

$$6\,(x+4)\,(x-1)$$

> $factor\left(\frac{x^3 - y^3}{x^4 - y^4}\right)$

$$\frac{x^2 + x\,y + y^2}{(y+x)\,\left(x^2 + y^2\right)}$$

> $factor\left(y^4 - 2\right)$

$$y^4 - 2$$

> $factor\left(y^4 - 2, \sqrt{2}\right)$

$$-\left(y^2 + \sqrt{2}\right)\,\left(-y^2 + \sqrt{2}\right)$$

Expanding parentheses : expand

expand (*expr*) – expansion of parentheses in an algebraic expression

```
> expand( (x + 1) (x + 2) )
```

A space or multiplication sign between brackets is needed!

$$x^2 + 3x + 2$$

```
> expand( sin(x + y) )
```

$$\sin(x)\cos(y) + \cos(x)\sin(y)$$

```
> expand( (x + 1) (y + z) )
```

$$xy + xz + y + z$$

```
> expand( (x + 1) (y + z), x + 1 )
```

$$(x + 1)y + (x + 1)z$$

Верно

```
> expand( (x + 1)·(x + 2) )
```

$$x^2 + 3x + 2$$

Неверно: между скобками нет пробела или знака умножения

```
> expand( (x + 1)(x + 2) )
```

$$x(x + 2) + 1$$

Bringing similar : collect

collect (*expr,var*) – casting similar expression members *expr* relative to the variable *var*

> $f := a\ln(x) - \ln(x)\,x - x$

$$f := a\ln(x) - \ln(x)\,x - x$$

> $collect(f, \ln(x))$

$$(a - x)\ln(x) - x$$

> $collect(f, x)$

$$(-\ln(x) - 1)\,x + a\ln(x)$$

> $f := a^3 x - x + a^3 + a$

$$f := a^3 x - x + a^3 + a$$

> $collect(f, x); collect(f, a^3)$

$$(a^3 - 1)\,x + a^3 + a$$

$$(x + 1)\,a^3 + a - x$$

> $collect(f, a)$

$$(x + 1)\,a^3 + a - x$$

Expression transformation : combine

combine (*expr*) – converts several terms into one
combine (*expr* , *param*) – converts several terms into one using parameters: **exp** , **trig** , **power** and others.

> $combine\left(\sin\left(\frac{1\,\pi}{8}\right)^4+\cos\left(3\,\pi\,\frac{1}{8}\right)^4+\sin\left(5\,\pi\,\frac{1}{8}\right)^4+\cos\left(7\,\pi\,\frac{1}{8}\right)^4\right)$

$$\frac{3}{2}$$

> $combine\left(4\sin(x)^3, trig\right)$

$$-\sin(3\,x)+3\sin(x)$$

> $combine\left(|x^2|\,|y+1|, \mathrm{abs}\right)$

$$|x^2\,(y+1)|$$

> $expr := 4\sin(x)^3\cdot a^x\cdot a^{3\,x}+e^y\,e^x$:

> $combine(expr, trig)$

$$-a^x\,a^{3\,x}\sin(3\,x)+3\,a^x\,a^{3\,x}\sin(x)+e^y\,e^x$$

> $combine(expr, \exp)$

$$4\sin(x)^3\,a^x\,a^{3\,x}+e^{y+x}$$

> $combine(\mathrm{expr}, \mathrm{power})$

$$4\sin(x)^3\,a^{4\,x}+e^{y+x}$$

Simplifying expressions : simplify

- **simplify** (*expr*) – simplifying an expression
- **simplify** (*expr* , *param*) – simplifies an expression using parameters: **exp** , **trig** , **power** and others.
- **simplify** (*expr* , ***assume*** *=prop*) – simplification of an expression under the assumption of the property prop

```
> eq := (cos(x) - sin(x)) * (cos(x) + sin(x)) : simplify(eq);
```

$$2\cos(x)^2 - 1$$

```
> g := sqrt(x^2); simplify(g, assume = positive)
```

$$g := \sqrt{x^2}$$

$$x$$

```
> f := sin(x)^2 + ln(2 x) + cos(x)^2 :
> simplify(f, trig); simplify(f, ln)
```

$$1 + \ln(2x)$$

$$\sin(x)^2 + \ln(2) + \ln(x) + \cos(x)^2$$

Сравнение *simplify* и *combine*

```
> f := 4 cos(a)^3 - 3 cos(a) :
> simplify(f); combine(f)
```

$$\cos(a)\left(4\cos(a)^2 - 3\right)$$

$$\cos(3a)$$