1. Solving algebraic equations

> *#1. Решение алгебраических уравнений*

> *# Решите алгебраическое уравнение с одним неизвестным. Выполните проверку сначала подставив поочередно корни в исходное уравнение, а затем попробуйте выполнить подстановку, использовав оператор $.*

> $restart:$

> $Digits := 12:$

> $f := x^3 + 2\cdot x^2 - x + 12;$

$$f := x^3 + 2x^2 - x + 12 \quad \textbf{(1.1)}$$

> $r := [solve(f, x)];$

$$r := \left[-\frac{\left(179 + 3\sqrt{3522}\right)^{1/3}}{3} - \frac{7}{3\left(179 + 3\sqrt{3522}\right)^{1/3}} - \frac{2}{3}, \frac{\left(179 + 3\sqrt{3522}\right)^{1/3}}{6} + \frac{7}{6\left(179 + 3\sqrt{3522}\right)^{1/3}} - \frac{2}{3} + \frac{I\sqrt{3}\left(-\frac{\left(179 + 3\sqrt{3522}\right)^{1/3}}{3} + \frac{7}{3\left(179 + 3\sqrt{3522}\right)^{1/3}}\right)}{2}, \frac{\left(179 + 3\sqrt{3522}\right)^{1/3}}{6} + \frac{7}{6\left(179 + 3\sqrt{3522}\right)^{1/3}} - \frac{2}{3} - \frac{I\sqrt{3}\left(-\frac{\left(179 + 3\sqrt{3522}\right)^{1/3}}{3} + \frac{7}{3\left(179 + 3\sqrt{3522}\right)^{1/3}}\right)}{2} \right] \quad \textbf{(1.2)}$$

> $fr := evalf(r);$

$$fr := [-3.36031619000, 0.680158095003 - 1.76308748579\, I, 0.680158095003 + 1.76308748579\, I] \quad \textbf{(1.3)}$$

> epsilon := 1e−5;

$$\epsilon := 0.00001 \quad \textbf{(1.4)}$$

> $subs(x = fr[1], f);$

$$-1. \times 10^{-10} \quad \textbf{(1.5)}$$

```
> is(subs(x=fr[1],f) < epsilon);
```
$$true \quad (1.6)$$

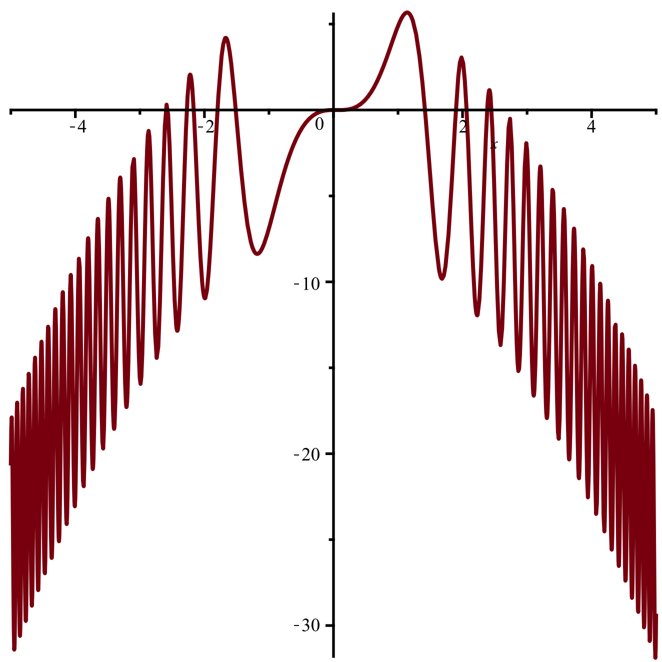
```
> is(subs(x=fr[1],f) < epsilon) $i=1..nops(fr);
```
$$true, true, true \quad (1.7)$$

2. Solving transcendental equations

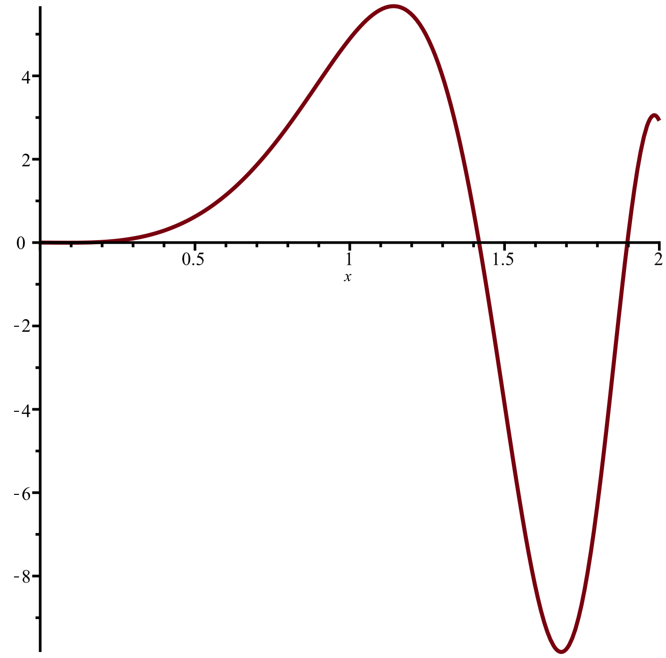
> # *Постройте график функции. Визуально определите корни на графике на отрезке [-2, 2]. Задайте значение Digits равным 5. Последовательно найдите все корни с помощью функции fsolve. Проверьте решение. Задайте значение Digits равным 12. Выполните заново поиск корней.*

```
> restart:
> Digits := 12;
```
$$Digits := 12 \quad (2.1)$$

```
> f := - x^2 + 7· sin(x^3);
```
$$f := -x^2 + 7\sin(x^3) \quad (2.2)$$

```
> plot(f, x=-5..5);
```



```
> plot(f, x=0..2);
```



> $r := solve(f, x);$

$$r := \frac{RootOf(-343\sin(\_Z)^3 + \_Z^2)}{7\sin(RootOf(-343\sin(\_Z)^3 + \_Z^2))} \qquad (2.3)$$

> $rr := [evalf(r)];$

$$rr := [\mathrm{Float(undefined)}] \qquad (2.4)$$

$$-1.797079804 \qquad (2.5)$$

> $xx[1] := fsolve(f, x = 0);$

$$xx_1 := 0. \qquad (2.6)$$

> $xx[2] := fsolve(f, x = 1.5);$

$$xx_2 := 1.41785053769 \qquad (2.7)$$

> $xx[3] := fsolve(f, x = 1.8);$

$$xx_3 := 1.89666216807 \qquad (2.8)$$

> $subs(x = xx[i], f)\$i = 1..3;$

$$0., -1.8 \times 10^{-10}, -1.7 \times 10^{-10} \qquad (2.9)$$

3. Solving Trigonometric Equations

> *#Решите тригонометрическое уравнение.*
> *Установите значение переменной _EnvAllSolutions равным true.*
> *Выполните поиск корней заново.*

> *restart* :

> $f := \sin(x);$

$$f := \sin(x) \qquad (3.1)$$

> $r := solve(f, x);$

$$r := 0 \quad (3.2)$$

> _EnvAllSolutions := true;

$$\_EnvAllSolutions := true \quad (3.3)$$

> r := solve(f, x);

$$r := \pi \_Z1{\sim} \quad (3.4)$$

> about(_Z1);

```
Originally _Z1, renamed _Z1~:
  is assumed to be: integer
```

> f := sin(x) + 2·cos(2·x);

$$f := \sin(x) + 2\cos(2x) \quad (3.5)$$

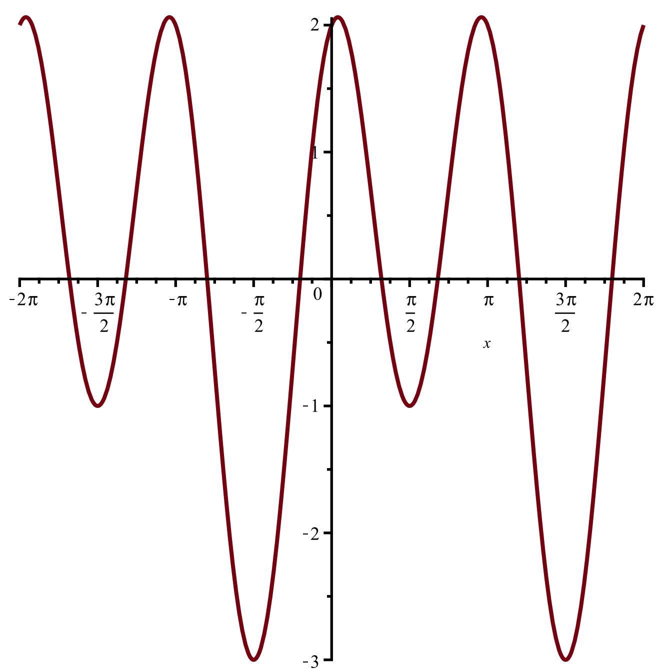
> r := solve(f, x);

$$r := \arctan\left(\frac{8\left(\frac{1}{8} - \frac{\sqrt{33}}{8}\right)}{\sqrt{30 + 2\sqrt{33}}}\right) + 2\pi\_Z2{\sim}, -\arctan\left(\frac{8\left(\frac{1}{8} - \frac{\sqrt{33}}{8}\right)}{\sqrt{30 + 2\sqrt{33}}}\right) - \pi + 2\pi\_Z2{\sim}, \arctan\left(\frac{8\left(\frac{1}{8} + \frac{\sqrt{33}}{8}\right)}{\sqrt{30 - 2\sqrt{33}}}\right) + 2\pi\_Z2{\sim}, -\arctan\left(\frac{8\left(\frac{1}{8} + \frac{\sqrt{33}}{8}\right)}{\sqrt{30 - 2\sqrt{33}}}\right) + \pi + 2\pi\_Z2{\sim} \quad (3.6)$$

> r := evalf[3](solve(f, x));

$$r := -0.635 + 6.28\_Z4{\sim}, -2.50 + 6.28\_Z4{\sim}, 1.00 + 6.28\_Z4{\sim}, 2.14 + 6.28\_Z4{\sim} \quad (3.7)$$

> plot(f, x);



4. Piecewise continuous function and its zeros

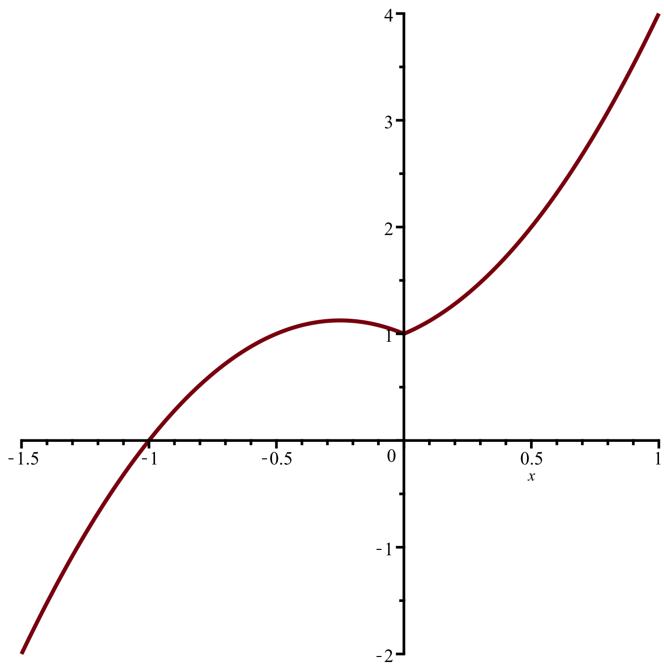
> #Кусочно-непрерывная функция и ее нули

> restart :

> $f := piecewise(x > 0, 2\cdot x^2 + x + 1, x \le 0, -2\cdot x^2 - x + 1);$

$$f := \begin{cases} 2x^2 + x + 1 & 0 < x \\ -2x^2 - x + 1 & x \le 0 \end{cases} \quad (4.1)$$

> $plot(f, x = -1.5 .. 1);$



> $r := solve(f, x);$

$$r := -1 \quad (4.2)$$

Integer Solutions

> #Посчитайте разницу между всеми корнями и целочисленными корнями

> restart :

> $f := (x^2 - 4)\cdot(x^3 - 7);$

$$f := (x^2 - 4)(x^3 - 7) \quad (5.1)$$

> $r := [solve(f, x)];$

$$r := \left[2, -2, 7^{1/3}, -\frac{7^{1/3}}{2} + \frac{I\sqrt{3}\,7^{1/3}}{2}, -\frac{7^{1/3}}{2} - \frac{I\sqrt{3}\,7^{1/3}}{2}\right] \quad (5.2)$$

> $ri := [isolve(f)];$

$$ri := [\{x = -2\}, \{x = 2\}] \quad (5.3)$$

> $nops(r) - nops(ri);$

$$3 \quad (5.4)$$

_EnvExplicit

> $restart:$

> $\_EnvExplicit := false;$

$$\_EnvExplicit := false \quad (6.1)$$

> $f := x^4 - 2 \cdot x^2 - 12\ ;$

$$f := x^4 - 2\,x^2 - 12 \quad (6.2)$$

> $r := solve(f, x);$

$$r := RootOf\left(\_Z^2 - 2\,RootOf\left(\_Z^2 - \_Z - 3\right)\right) \quad (6.3)$$

> $\_EnvExplicit := true;$

$$\_EnvExplicit := true \quad (6.4)$$

> $f := x^4 - 2 \cdot x^2 - 12\ ;$

$$f := x^4 - 2\,x^2 - 12 \quad (6.5)$$

> $r := solve(f, x);$

$$r := \mathrm{I}\sqrt{-1 + \sqrt{13}},\ -\mathrm{I}\sqrt{-1 + \sqrt{13}},\ \sqrt{1 + \sqrt{13}},\ -\sqrt{1 + \sqrt{13}} \quad (6.6)$$

Solving systems of equations

> #Решение систем уравнений

> $restart:$

> $f1 := x^2 - y^2 = 2; f2 := 2\,x^2 \cdot y - x^2 + 12;$

$$f1 := x^2 - y^2 = 2$$

$$f2 := 2\,x^2\,y - x^2 + 12 \quad (7.1)$$

> $r := solve(\{f1, f2\}, \{x, y\});$

$$r := \left\{x = RootOf\left(4\,\_Z^6 - 9\,\_Z^4 + 24\,\_Z^2 - 144\right), y = -\frac{RootOf\left(4\,\_Z^6 - 9\,\_Z^4 + 24\,\_Z^2 - 144\right)^4}{6} + \frac{3\,RootOf\left(4\,\_Z^6 - 9\,\_Z^4 + 24\,\_Z^2 - 144\right)^2}{8} - \frac{1}{2}\right\} \quad (7.2)$$

> $evalf(r);$

$$\{x = 1.867721810, y = -1.219993754\} \quad (7.3)$$

> $\_EnvExplicit := true;$

$$\_EnvExplicit := true \quad (7.4)$$

> $r := solve(\{f1, f2\}, \{x, y\}):$

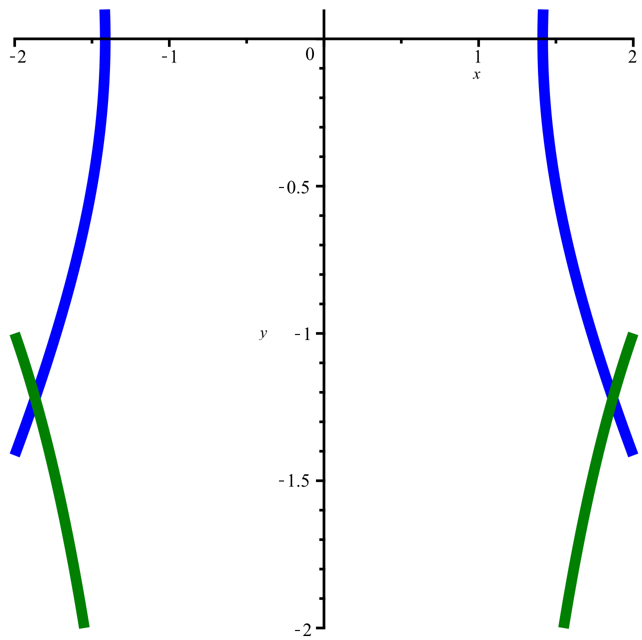
> $r := evalf(solve(\{f1, f2\}, \{x, y\}));$

$$r := \{x = 1.867721810, y = -1.219993754\}, \{x = -1.867721810, y = -1.219993754\}, \{x = 1.138700459 + 1.384135512\,\mathrm{I}, y = 0.8599968780 + 1.832699378\,\mathrm{I}\}, \{x \quad (7.5)$$

$= -1.138700459 - 1.384135512\,I, y = 0.8599968780 + 1.832699378\,I\}, \{x$
$= 1.138700459 - 1.384135512\,I, y = 0.8599968780 - 1.832699378\,I\}, \{x$
$= -1.138700459 + 1.384135512\,I, y = 0.8599968780 - 1.832699378\,I\}$

> *with*(*plots*) :

> *implicitplot*([*f1*, *f2*], *x* =-2 ..2, *y* =-2 ..0.1, *color* = ["Blue", "Green"], *thickness* = 4);



Solving inequalities

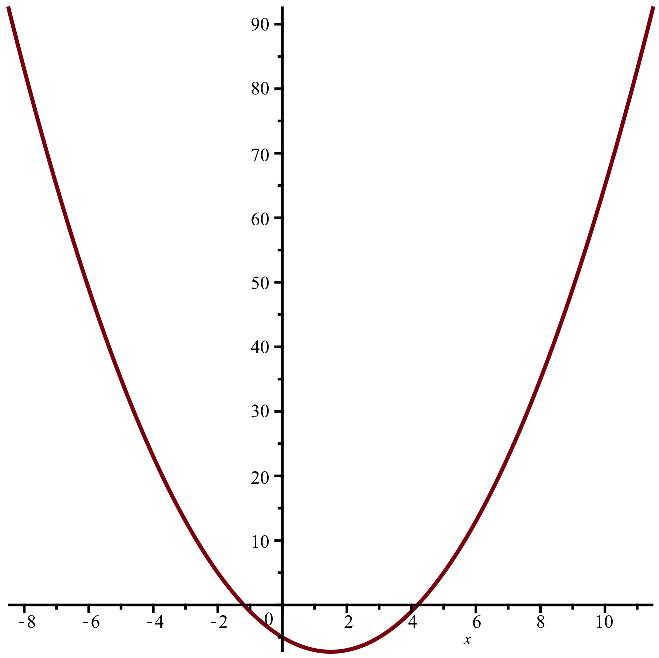
> #Решение неравенств

> *restart* :

> $f := x^2 - 3\cdot x - 5;$

$$f := x^2 - 3\,x - 5 \qquad \textbf{(8.1)}$$

> *plot*(*f*, *x*);



> $r := solve(f < 0, x);$

$$r := \left(\frac{3}{2} - \frac{\sqrt{29}}{2}, \frac{3}{2} + \frac{\sqrt{29}}{2}\right) \quad \textbf{(8.2)}$$

> $with(RootFinding);$

[*Analytic, AnalyticZerosFound, BivariatePolynomial, EnclosingBox, HasRealRoots, Homotopy, Isolate, NextZero, Parametric, WitnessPoints*] **(8.3)**

HasRealRoots

> $HasRealRoots([x^2 + y^2 + 2]);$

false **(8.4)**

> $solve([x^2 + y^2 + 2], [x, y]);$

$$\left[\left[x = \sqrt{-y^2 - 2}, y = y\right], \left[x = -\sqrt{-y^2 - 2}, y = y\right]\right] \quad \textbf{(8.5)}$$

> $HasRealRoots([x^4 - 4x^2 + 4 = 0]);$

true **(8.6)**

> $solve([x^4 - 4x^2 + 4 = 0]);$

$$\{x = \sqrt{2}\}, \{x = -\sqrt{2}\}, \{x = \sqrt{2}\}, \{x = -\sqrt{2}\} \quad \textbf{(8.7)}$$

NextZero

> $r0 := NextZero(x \mapsto \sin(x), 0)$

$$r0 := 3.141592653 \quad \textbf{(8.8)}$$

> $r0 := NextZero(x \mapsto \sin(x), r0)$

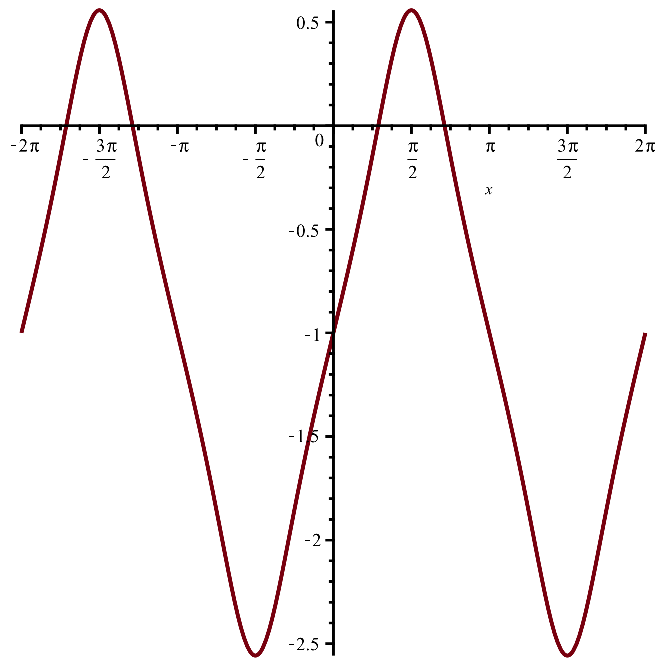
$$r0 := 6.283185307 \quad \textbf{(8.9)}$$

AnalyticZerosFound

> $f := \tan(\sin(x)) - 1;$

$$f := \tan(\sin(x)) - 1 \quad \textbf{(8.10)}$$

> $plot(f);$



> $evalf(solve(f, x));$

$$0.9033391109 \quad \textbf{(8.11)}$$

> $Analytic\left(f, x, -\frac{I}{2}..1+I\right);$

$$0.903339110766515 \quad \textbf{(8.12)}$$

> $g := 23\,x^5 + 105\,x^4 - 10\,x^2 + 17\,x;$

$$g := 23\,x^5 + 105\,x^4 - 10\,x^2 + 17\,x \quad \textbf{(8.13)}$$

> $evalf(solve(g, x));$

$0., 0.3040664543 + 0.4040619058\,I, -0.6371813185, -4.536168981, 0.3040664543 - 0.4040619058\,I$ **(8.14)**

> $Analytic(g, x, re = -5..1, im = -1..1);$

$0. + 0.\,I, -0.637181318531050, 0.304066454284907 - 0.404061905751759\,I, 0.304066454284907 + 0.404061905751759\,I, -4.53616898134312$ **(8.15)**

$with(RootFinding[Parametric]);$

$[CellDecomposition, CellDescription, CellLocation, CellPlot, CellsWithSolutions, DiscriminantVariety, NumberOfSolutions, SampleSolutions]$ **(8.16)**

> $f := x^2 + a \cdot x + b;$

$$f := a\,x + x^2 + b \quad \textbf{(8.17)}$$

> $DiscriminantVariety([f=0], [x])$

$$[[a^2 - 4\,b]] \quad \textbf{(8.18)}$$

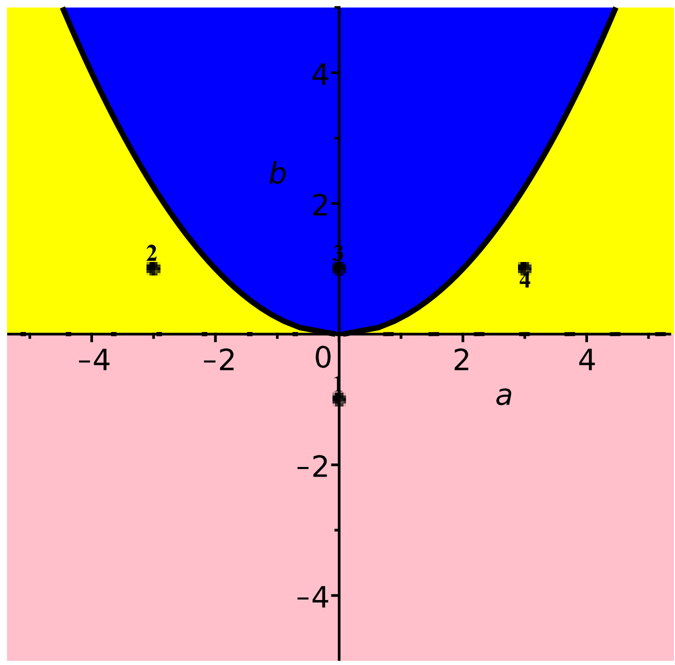
> $discrim(f, x);$

$$a^2 - 4\,b \quad \textbf{(8.19)}$$

> $m := CellDecomposition([f=0], [x]);$

(8.20)

> $CellPlot(m, 'samplepoints', labelfont = ["HELVETICA", 18], axesfont = ["HELVETICA", "ROMAN", 18]);$



> $f1 := subs(a=0, b=1, f);$

$$f1 := x^2 + 1 \quad \textbf{(8.21)}$$

> $solve(f1, x);$

$$I, -I \quad \textbf{(8.22)}$$

Solving systems of inequalities

> #Решение систем неравенств

> $restart:$

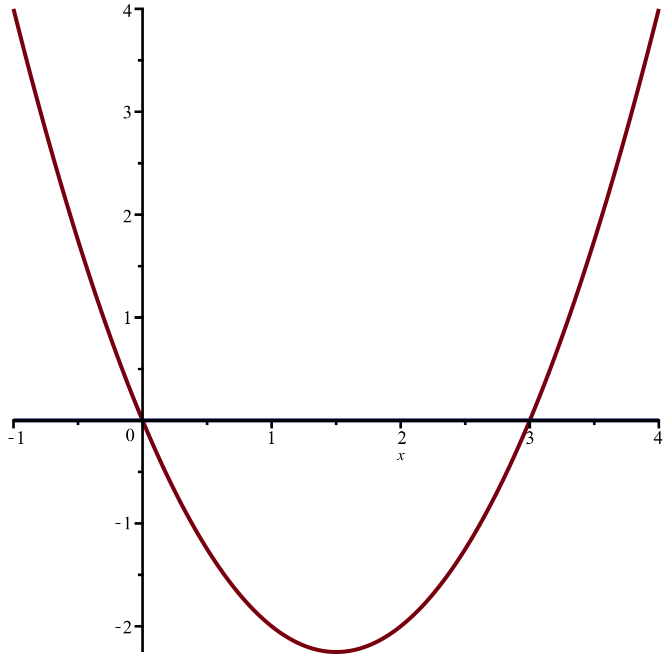
> $f1 := x^2 - 3\cdot x - 5 < 0;$

$$f1 := x^2 - 3x < 5 \quad \textbf{(9.1)}$$

> $f2 := x^{\frac{1}{2}} - 2\cdot x + 4 > 0;$

$$f2 := 0 < \sqrt{x} - 2x + 4 \quad \textbf{(9.2)}$$

> $plot([lhs(f1), lhs(f2)], x);$



> $solve([f1, f2], x);$

$$\left\{0 \le x, x < \frac{17}{8} + \frac{\sqrt{33}}{8}\right\} \qquad \textbf{(9.3)}$$

> $evalf(solve([f1, f2], x));$

$$\{0. \le x, x < 2.843070331\} \qquad \textbf{(9.4)}$$