1. Solving algebraic equations

> *#1. Решение алгебраических уравнений*

> *# Решите алгебраическое уравнение с одним неизвестным. Выполните проверку сначала подставив поочередно корни в исходное уравнение, а затем попробуйте выполнить подставновку, использовав оператор $.*

> *restart* :

> *Digits* := 12 :

> $f := x^3 + 2 \cdot x^2 - x + 12;$

$$f := x^3 + 2\,x^2 - x + 12 \tag{1.1}$$

> $r := [solve(f, x)];$

$$r := \left[-\frac{(179 + 3\sqrt{3522})^{1/3}}{3} - \frac{7}{3\,(179 + 3\sqrt{3522})^{1/3}} - \frac{2}{3}, \right. \tag{1.2}$$

$$\frac{(179 + 3\sqrt{3522})^{1/3}}{6} + \frac{7}{6\,(179 + 3\sqrt{3522})^{1/3}} - \frac{2}{3} + \frac{\mathrm{I}\sqrt{3}\left(-\frac{(179 + 3\sqrt{3522})^{1/3}}{3} + \frac{7}{3\,(179 + 3\sqrt{3522})^{1/3}}\right)}{2},$$

$$\left. \frac{(179 + 3\sqrt{3522})^{1/3}}{6} + \frac{7}{6\,(179 + 3\sqrt{3522})^{1/3}} - \frac{2}{3} - \frac{\mathrm{I}\sqrt{3}\left(-\frac{(179 + 3\sqrt{3522})^{1/3}}{3} + \frac{7}{3\,(179 + 3\sqrt{3522})^{1/3}}\right)}{2} \right]$$

> $fr := evalf(r);$

$$fr := [-3.36031619000, 0.680158095003 - 1.76308748579\,\mathrm{I}, 0.680158095003 + 1.76308748579\,\mathrm{I}] \tag{1.3}$$

> epsilon := 1e−5;

$$\epsilon := 0.00001 \tag{1.4}$$

> $subs(x = fr[1], f);$

$$-1.\,10^{-10} \tag{1.5}$$

> $is(\mathrm{abs}(subs(x = fr[1], f)) < \mathrm{epsilon});$

$$true \tag{1.6}$$

> $is(\mathrm{abs}(subs(x = fr[2], f)) < \mathrm{epsilon});$

$$true \quad (1.7)$$

> $is(\text{abs}(subs(x=fr[3], f)) < \text{epsilon});$

$$true \quad (1.8)$$

> $r := [\text{abs}(subs(x=fr[i], f)) \ \$ i=1..3];$

$$r := [1.\ 10^{-10}, 1.1\ 10^{-10}, 1.1\ 10^{-10}] \quad (1.9)$$

> $map(x \rightarrow x < \text{epsilon}, r)$

$$[1.\ 10^{-10} < 0.00001, 1.1\ 10^{-10} < 0.00001, 1.1\ 10^{-10} < 0.00001] \quad (1.10)$$

> $map(x \rightarrow is(x < \text{epsilon}), r)$

$$[true, true, true] \quad (1.11)$$

2. Solving transcendental equations

> *# Постройте график функции. Визуально определите корни на графике на отрезке [-2, 2]. Задайте значение Digits равным 5. Последовательно найдите все корни с помощью функции fsolve. Проверьте решение. Задайте значение Digits равным 12. Выполните заново поиск корней.*

> $restart:$

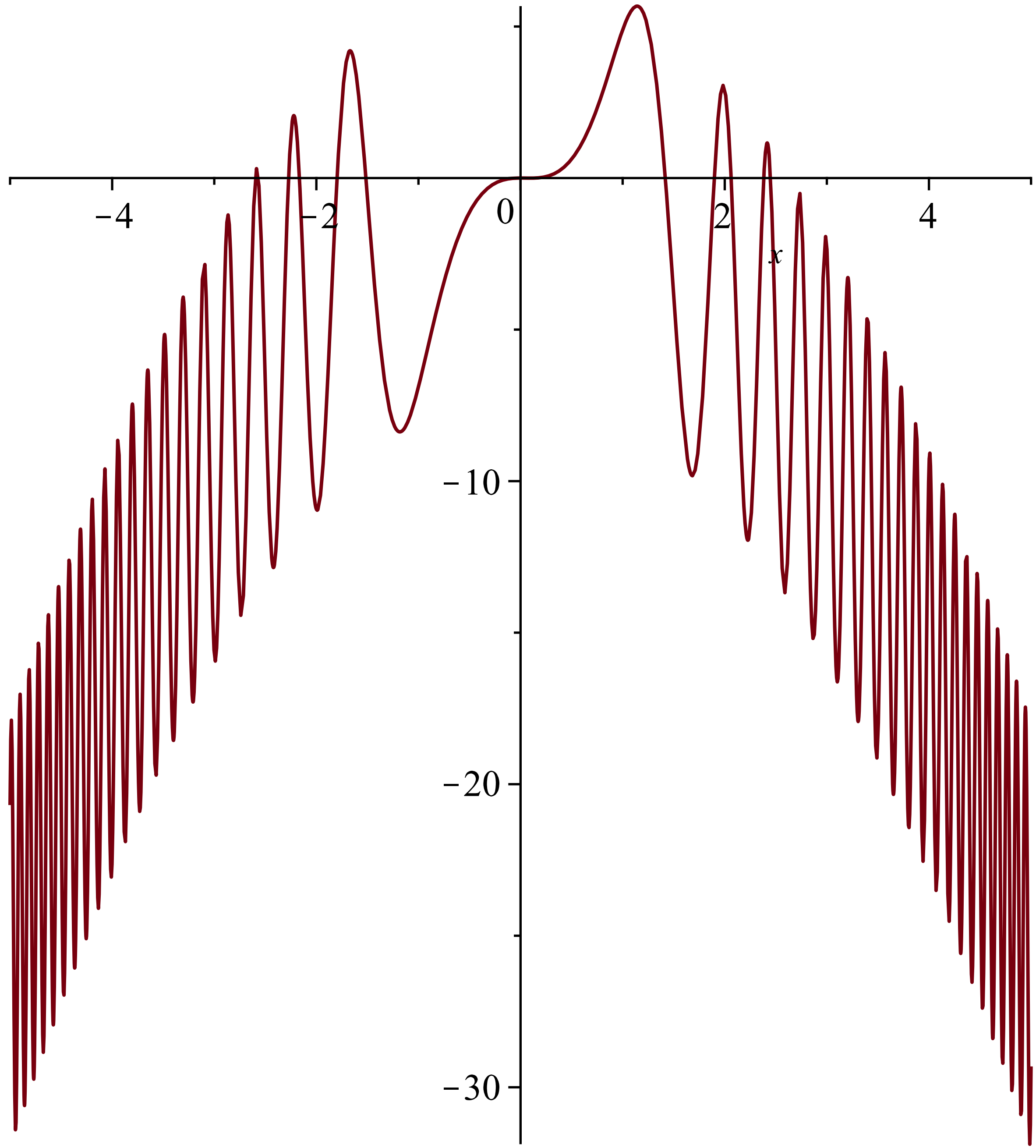
> $Digits := 12;$

$$Digits := 12 \quad (2.1)$$

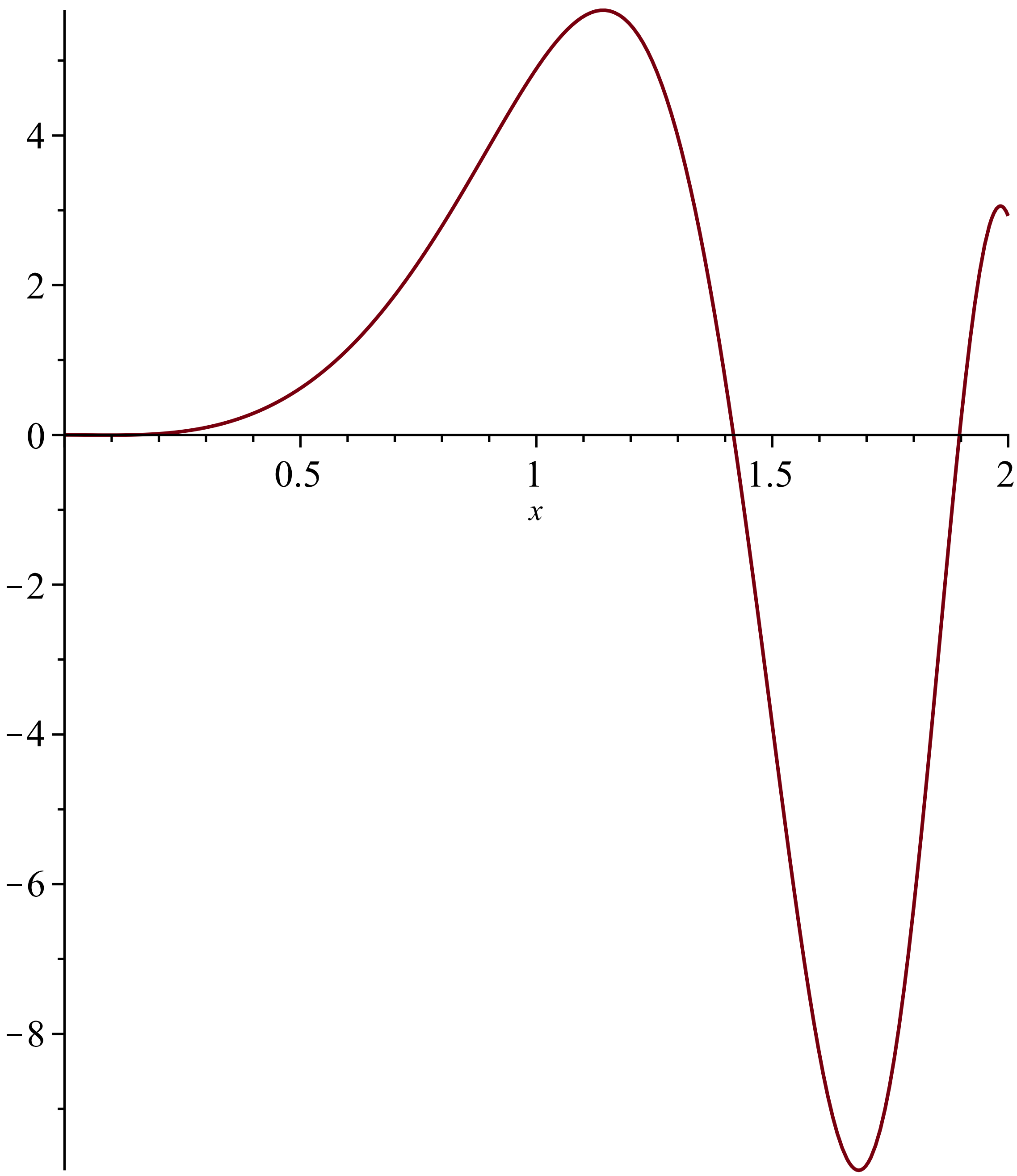
> $f := -x^2 + 7 \cdot \sin(x^3);$

$$f := -x^2 + 7 \sin(x^3) \quad (2.2)$$

> $plot(f, x=-5..5);$



> $plot(f, x=0..2);$



> $r := solve(f, x);$

$$r := \frac{RootOf(-343\sin(\_Z)^3 + \_Z^2)}{7\sin(RootOf(-343\sin(\_Z)^3 + \_Z^2))} \qquad \textbf{(2.3)}$$

> $rr := [evalf(r)];$

$$rr := [\text{Float(undefined)}] \qquad \textbf{(2.4)}$$

$$-1.797079804 \qquad \textbf{(2.5)}$$

> $xx[1] := fsolve(f, x = 0);$

$$xx_1 := 0. \qquad \textbf{(2.6)}$$

> $xx[2] := fsolve(f, x = 1.5);$

$$xx_2 := 1.41785053769 \qquad \textbf{(2.7)}$$

> $xx[3] := fsolve(f, x = 1.8);$

$$xx_3 := 1.89666216807 \quad \textbf{(2.8)}$$

> _EnvExplicit := true :

> r := solve(f, x);

$$r := \frac{RootOf(-343\sin(\_Z)^3 + \_Z^2)}{7\sin(RootOf(-343\sin(\_Z)^3 + \_Z^2))} \quad \textbf{(2.9)}$$

> rr := convert(r, radical);

$$rr := \frac{RootOf(-343\sin(\_Z)^3 + \_Z^2)}{7\sin(RootOf(-343\sin(\_Z)^3 + \_Z^2))} \quad \textbf{(2.10)}$$

> subs(x = xx[i], f)$i = 1 ..3;

$$0., -1.8 \times 10^{-10}, -1.7 \times 10^{-10} \quad \textbf{(2.11)}$$

3. Solving Trigonometric Equations

> #Решите тригонометрическое уравнение.
Установите значение переменной _EnvAllSolutions равным true.
Выполните поиск корней заново.

> restart :

> f := sin(x);

$$f := \sin(x) \quad \textbf{(3.1)}$$

> r := solve(f, x);

$$r := 0 \quad \textbf{(3.2)}$$

> _EnvAllSolutions := true;

$$\_EnvAllSolutions := true \quad \textbf{(3.3)}$$

> r := solve(f, x);

$$r := \pi\_Z1\sim \quad \textbf{(3.4)}$$

> about(_Z1);

```
Originally _Z1, renamed _Z1~:
  is assumed to be: integer
```

> f := sin(x) + 2·cos(2·x);

$$f := \sin(x) + 2\cos(2x) \quad \textbf{(3.5)}$$

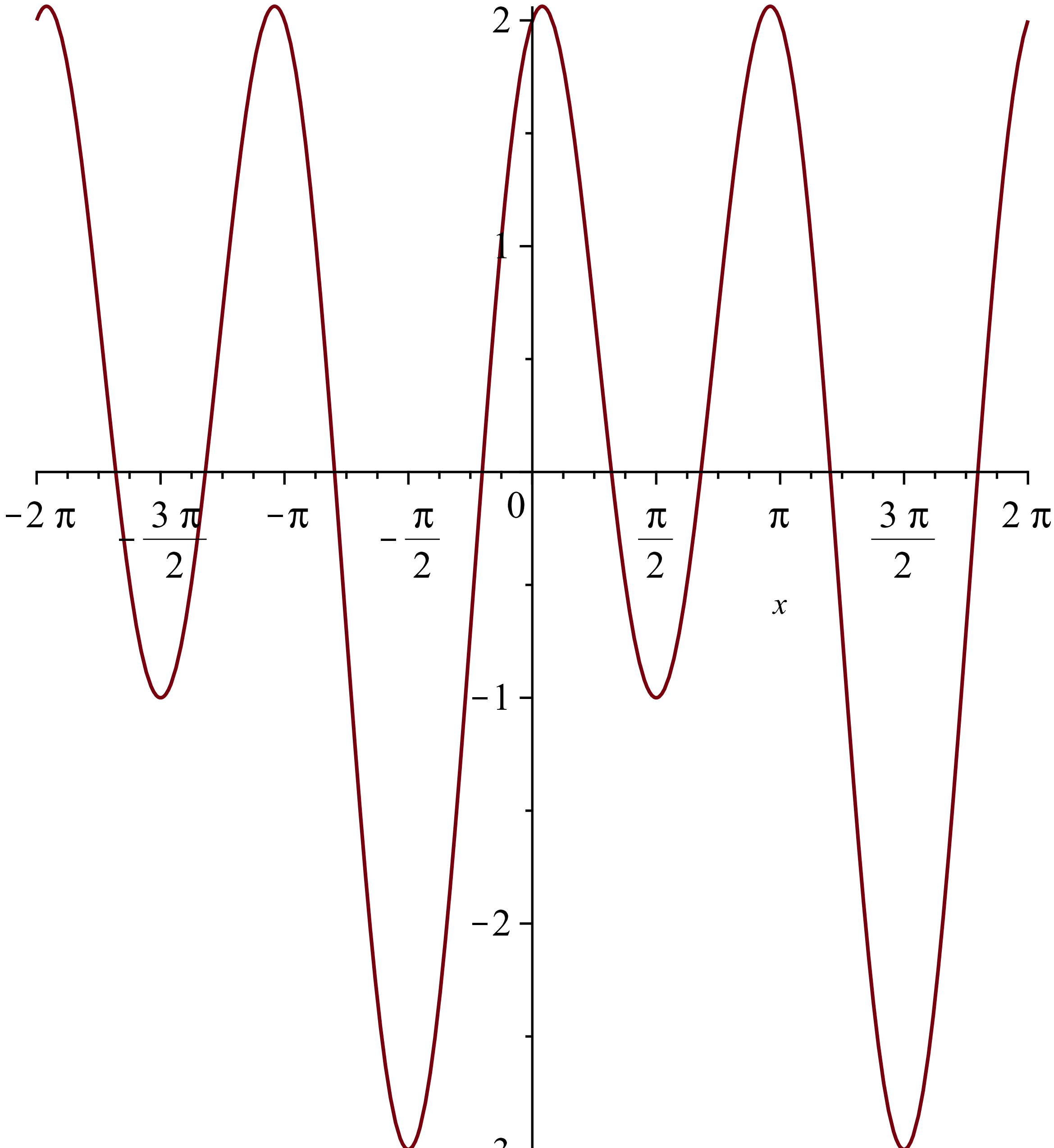
> r := solve(f, x);

$$r := \arctan\left(\frac{8\left(\frac{1}{8} - \frac{\sqrt{33}}{8}\right)}{\sqrt{30 + 2\sqrt{33}}}\right) + 2\pi\_Z2\sim, -\arctan\left(\frac{8\left(\frac{1}{8} - \frac{\sqrt{33}}{8}\right)}{\sqrt{30 + 2\sqrt{33}}}\right) - \pi + 2\pi\_Z2\sim, \arctan\left(\frac{8\left(\frac{1}{8} + \frac{\sqrt{33}}{8}\right)}{\sqrt{30 - 2\sqrt{33}}}\right) + 2\pi\_Z2\sim, -\arctan\left(\frac{8\left(\frac{1}{8} + \frac{\sqrt{33}}{8}\right)}{\sqrt{30 - 2\sqrt{33}}}\right) + \pi + 2\pi\_Z2\sim \quad \textbf{(3.6)}$$

> r := evalf[3](solve(f, x));

$r := -0.635 + 6.28\_Z4\sim, -2.50 + 6.28\_Z4\sim, 1.00 + 6.28\_Z4\sim, 2.14 + 6.28\_Z4\sim$ **(3.7)**

> *plot*(*f*, *x*);



4. Piecewise continuous function and its zeros

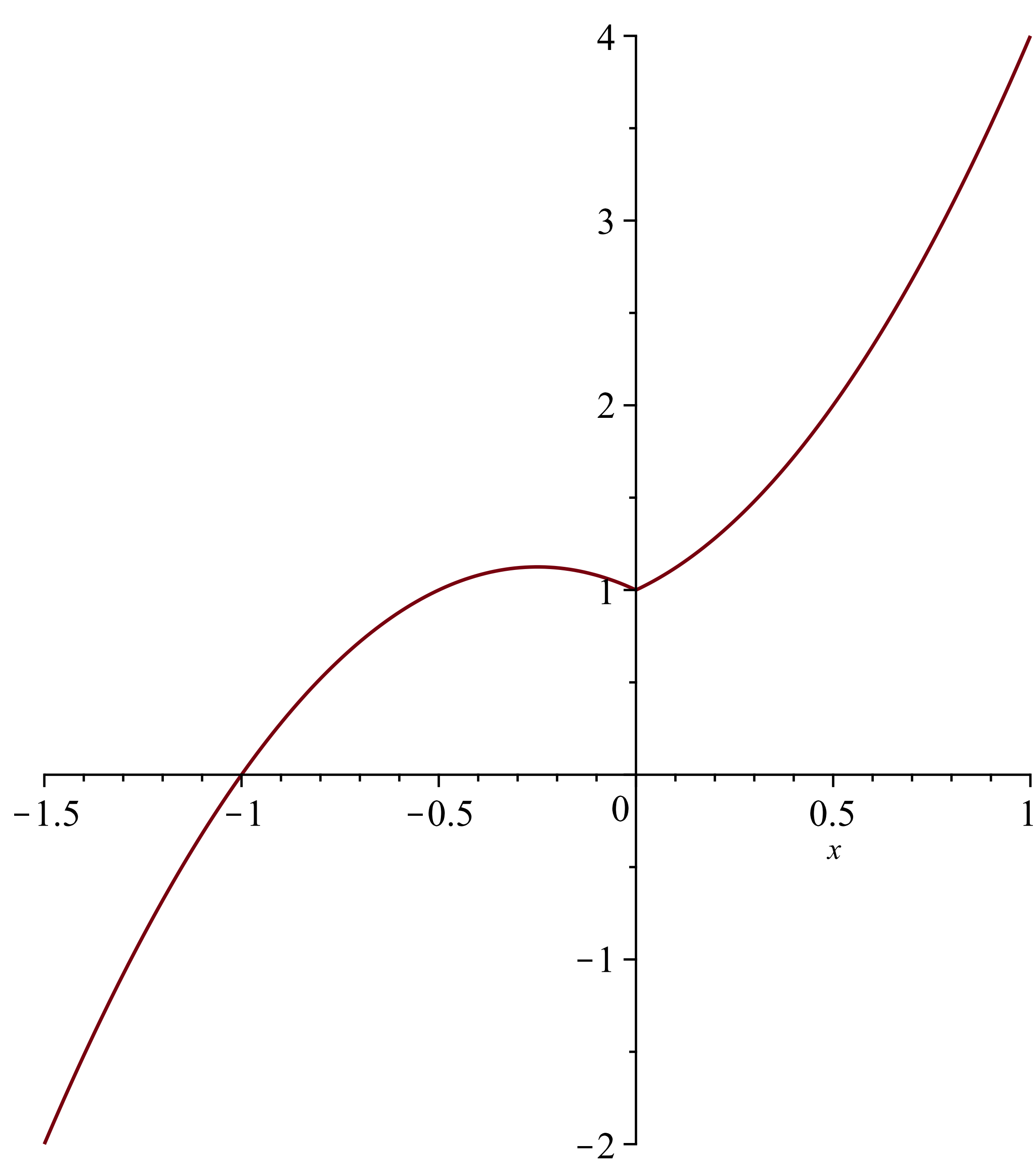
> *#Кусочно-непрерывная функция и ее нули*

> *restart* :

> $f := piecewise(x > 0, 2\cdot x^2 + x + 1, x \le 0, -2\cdot x^2 - x + 1);$

$$f := \begin{cases} 2x^2 + x + 1 & 0 < x \\ -2x^2 - x + 1 & x \le 0 \end{cases} \quad \textbf{(4.1)}$$

> *plot*(*f*, *x* =-1.5 ..1);



> $r := solve(f, x);$

$$r := -1 \quad \textbf{(4.2)}$$

Integer Solutions

> *#Посчитайте разницу между всеми корнями и целочисленными корнями*

> $restart:$

> $f := (x^2 - 4) \cdot (x^3 - 7);$

$$f := (x^2 - 4)\,(x^3 - 7) \quad \textbf{(5.1)}$$

> $r := [solve(f, x)];$

(5.2)

$$r := \left[2, -2, 7^{1/3}, -\frac{7^{1/3}}{2} + \frac{I\sqrt{3}\,7^{1/3}}{2}, -\frac{7^{1/3}}{2} - \frac{I\sqrt{3}\,7^{1/3}}{2}\right] \quad (5.2)$$

```
> ri := [isolve(f)];
```

$$ri := [\{x=-2\}, \{x=2\}] \quad (5.3)$$

```
> nops(r) - nops(ri);
```

$$3 \quad (5.4)$$

_EnvExplicit

```
> restart :
> _EnvExplicit := false;
```

$$\_EnvExplicit := false \quad (6.1)$$

```
> f := x^4 - 2·x^2 - 12 ;
```

$$f := x^4 - 2x^2 - 12 \quad (6.2)$$

```
> r := solve(f, x);
```

$$r := RootOf(\_Z^2 - 2\,RootOf(\_Z^2 - \_Z - 3)) \quad (6.3)$$

```
> _EnvExplicit := true;
```

$$\_EnvExplicit := true \quad (6.4)$$

```
> f := x^4 - 2·x^2 - 12 ;
```

$$f := x^4 - 2x^2 - 12 \quad (6.5)$$

```
> r := solve(f, x);
```

$$r := I\sqrt{-1+\sqrt{13}}, -I\sqrt{-1+\sqrt{13}}, \sqrt{1+\sqrt{13}}, -\sqrt{1+\sqrt{13}} \quad (6.6)$$

Solving systems of equations

```
> #Решение систем уравнений
> restart :
> f1 := x^2 - y^2 = 2; f2 := 2 x^2·y - x^2 + 12;
```

$$f1 := x^2 - y^2 = 2$$

$$f2 := 2x^2y - x^2 + 12 \quad (7.1)$$

```
> r := solve({f1, f2}, {x, y});
```

$$r := \left\{x = RootOf(4\_Z^6 - 9\_Z^4 + 24\_Z^2 - 144), y = -\frac{RootOf(4\_Z^6 - 9\_Z^4 + 24\_Z^2 - 144)^4}{6} + \frac{3\,RootOf(4\_Z^6 - 9\_Z^4 + 24\_Z^2 - 144)^2}{8} - \frac{1}{2}\right\} \quad (7.2)$$

```
> evalf(r);
```

$$\{x = 1.867721810, y = -1.219993754\} \quad (7.3)$$

```
> _EnvExplicit := true;
```

$$EnvExplicit := true \quad (7.4)$$

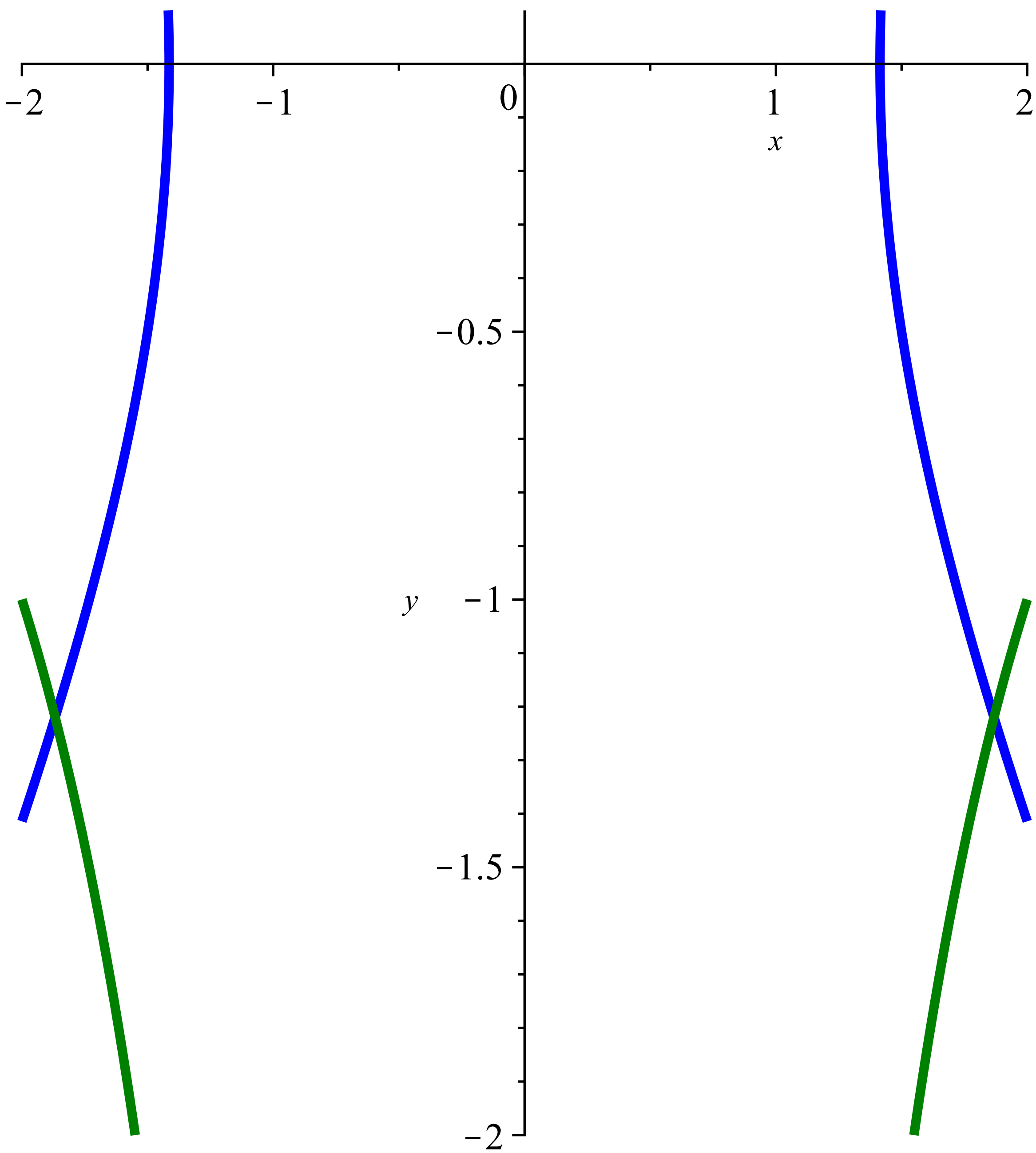
> $r := solve(\{f1, f2\}, \{x, y\})$:

> $r := evalf(solve(\{f1, f2\}, \{x, y\}))$;

$r := \{x = 1.867721810, y = -1.219993754\}, \{x = -1.867721810, y = -1.219993754\}, \{x = 1.138700459 + 1.384135512\,I, y = 0.8599968780 + 1.832699378\,I\}, \{x = -1.138700459 - 1.384135512\,I, y = 0.8599968780 + 1.832699378\,I\}, \{x = 1.138700459 - 1.384135512\,I, y = 0.8599968780 - 1.832699378\,I\}, \{x = -1.138700459 + 1.384135512\,I, y = 0.8599968780 - 1.832699378\,I\}$ **(7.5)**

> $with(plots)$:

> $implicitplot([f1, f2], x = -2..2, y = -2..0.1, color = ["Blue", "Green"], thickness = 4)$;



Solving inequalities

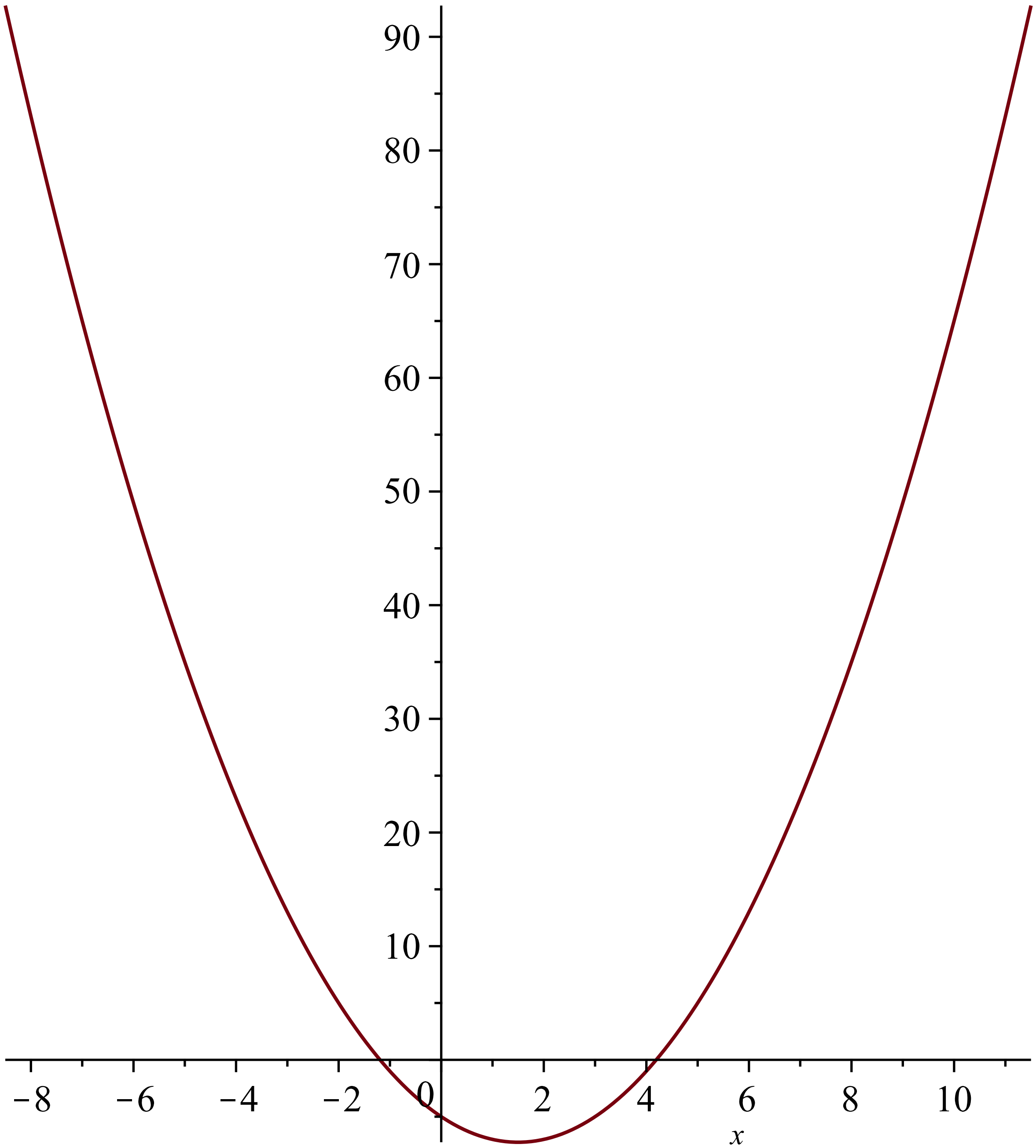
> #Решение неравенств

> restart :

> $f := x^2 - 3\cdot x - 5;$

$$f := x^2 - 3x - 5 \quad (8.1)$$

> $plot(f, x);$



> $r := solve(f < 0, x);$

$$r := \left(\frac{3}{2} - \frac{\sqrt{29}}{2}, \frac{3}{2} + \frac{\sqrt{29}}{2}\right) \quad (8.2)$$

> with(RootFinding);

[Analytic, AnalyticZerosFound, BivariatePolynomial, EnclosingBox, HasRealRoots, Homotopy, Isolate, NextZero, Parametric, WitnessPoints] **(8.3)**

HasRealRoots

> $HasRealRoots([x^2+y^2+2]);$

$$false \qquad (8.4)$$

> $solve([x^2+y^2+2], [x, y]);$

$$\left[\left[x=\sqrt{-y^2-2}, y=y\right], \left[x=-\sqrt{-y^2-2}, y=y\right]\right] \qquad (8.5)$$

> $HasRealRoots([x^4-4x^2+4=0]);$

$$true \qquad (8.6)$$

> $solve([x^4-4x^2+4=0]);$

$$\{x=\sqrt{2}\}, \{x=-\sqrt{2}\}, \{x=\sqrt{2}\}, \{x=-\sqrt{2}\} \qquad (8.7)$$

NextZero

> $r0 := NextZero(x \mapsto \sin(x), 0)$

$$r0 := 3.141592653 \qquad (8.8)$$

> $r0 := NextZero(x \mapsto \sin(x), r0)$

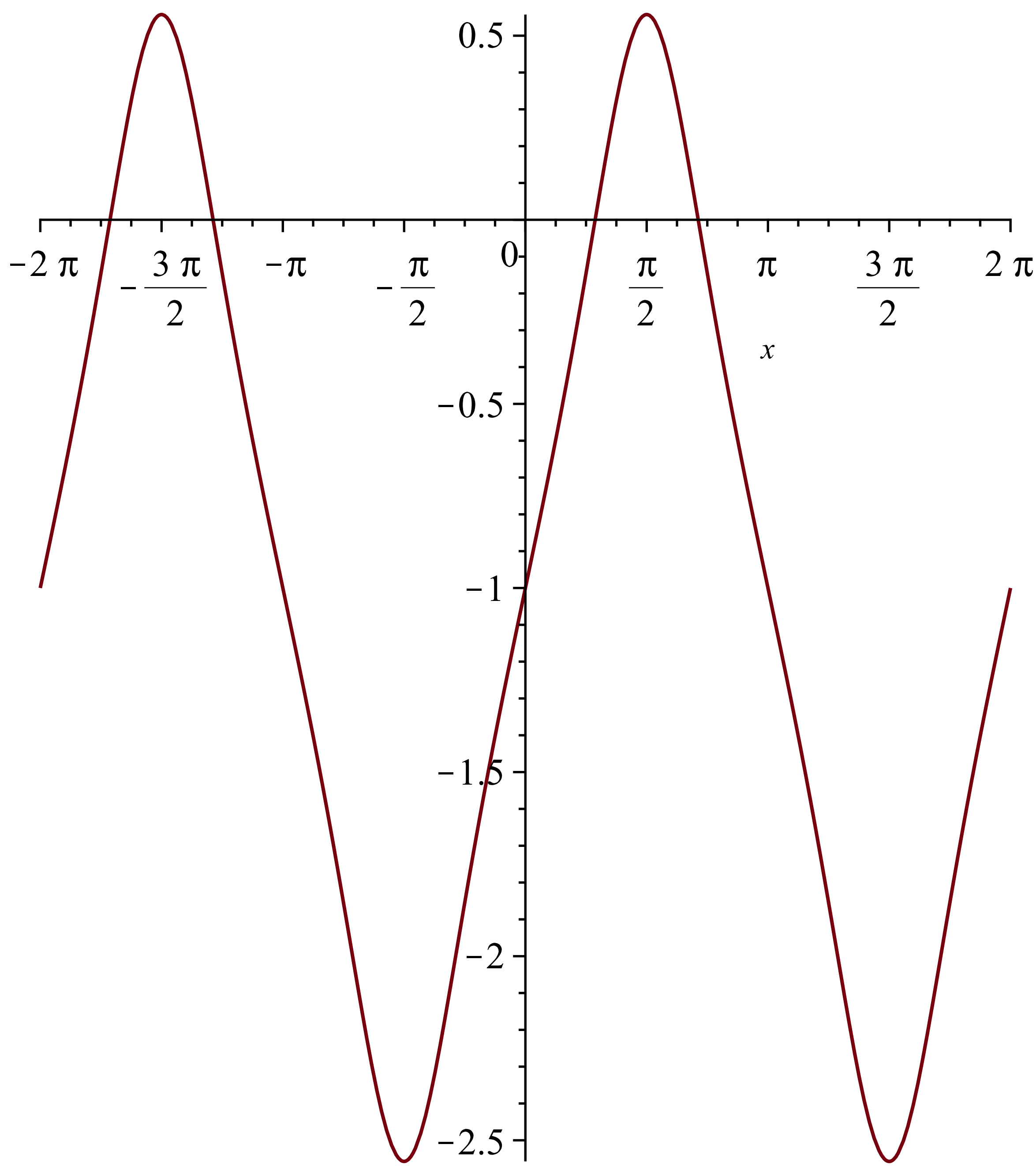
$$r0 := 6.283185307 \qquad (8.9)$$

AnalyticZerosFound

> $f := \tan(\sin(x)) - 1;$

$$f := \tan(\sin(x)) - 1 \qquad (8.10)$$

> $plot(f);$



> *evalf*(*solve*(*f*, *x*));

$$0.9033391109 \quad \textbf{(8.11)}$$

> *Analytic*$\left(f, x, -\frac{I}{2} .. 1 + I\right)$;

$$0.903339110766515 \quad \textbf{(8.12)}$$

> $g := 23\,x^5 + 105\,x^4 - 10\,x^2 + 17\,x$;

$$g := 23\,x^5 + 105\,x^4 - 10\,x^2 + 17\,x \quad \textbf{(8.13)}$$

> *evalf*(*solve*(*g*, *x*));

$$0., 0.3040664543 + 0.4040619058\,I, -0.6371813185, -4.536168981, 0.3040664543 - 0.4040619058\,I \quad \textbf{(8.14)}$$

> *Analytic*(*g*, *x*, *re* = −5 ..1, *im* = −1 ..1);

(8.15)

$0. + 0. I, -0.637181318531050, 0.304066454284907 - 0.404061905751759 I,$ **(8.15)**
$0.304066454284907 + 0.404061905751759 I, -4.53616898134312$

with(*RootFinding*[*Parametric*]);

[*CellDecomposition*, *CellDescription*, *CellLocation*, *CellPlot*, *CellsWithSolutions*, *DiscriminantVariety*, *NumberOfSolutions*, *SampleSolutions*] **(8.16)**

> $f := x^2 + a \cdot x + b;$

$$f := a x + x^2 + b \quad \textbf{(8.17)}$$

> *DiscriminantVariety*([$f = 0$], [x])

$$[[a^2 - 4 b]] \quad \textbf{(8.18)}$$

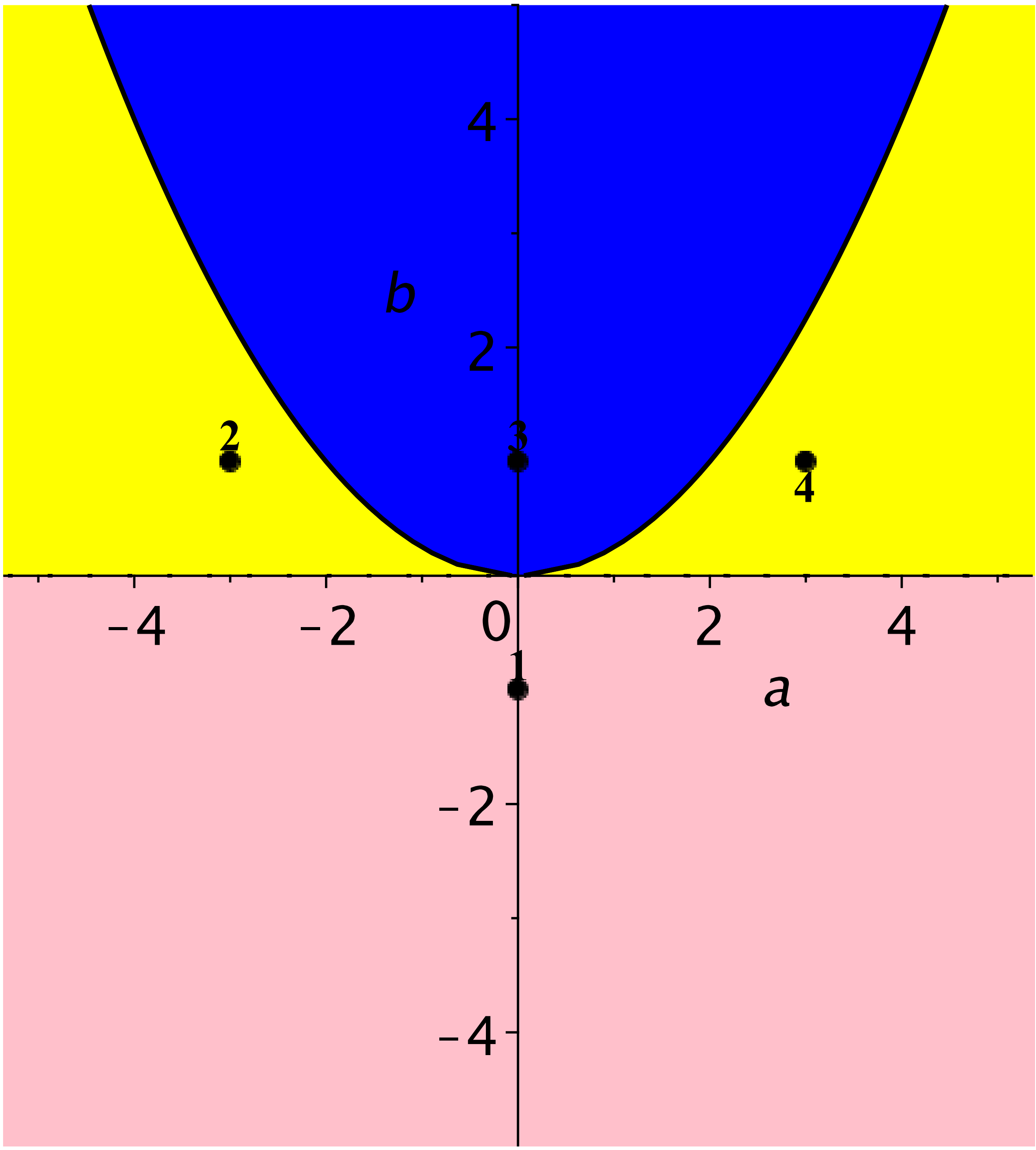
> *discrim*(f, x);

$$a^2 - 4 b \quad \textbf{(8.19)}$$

> $m :=$ *CellDecomposition*([$f = 0$], [x]);

(8.20)

> *CellPlot*(m, '*samplepoints*', *labelfont* = ["HELVETICA", 18], *axesfont* = ["HELVETICA", "ROMAN", 18]);



> $f1 := subs(a=0, b=1, f);$

$$f1 := x^2 + 1 \qquad (8.21)$$

> $solve(f1, x);$

$$I, -I \qquad (8.22)$$

Solving systems of inequalities

> #Решение систем неравенств

> $restart:$

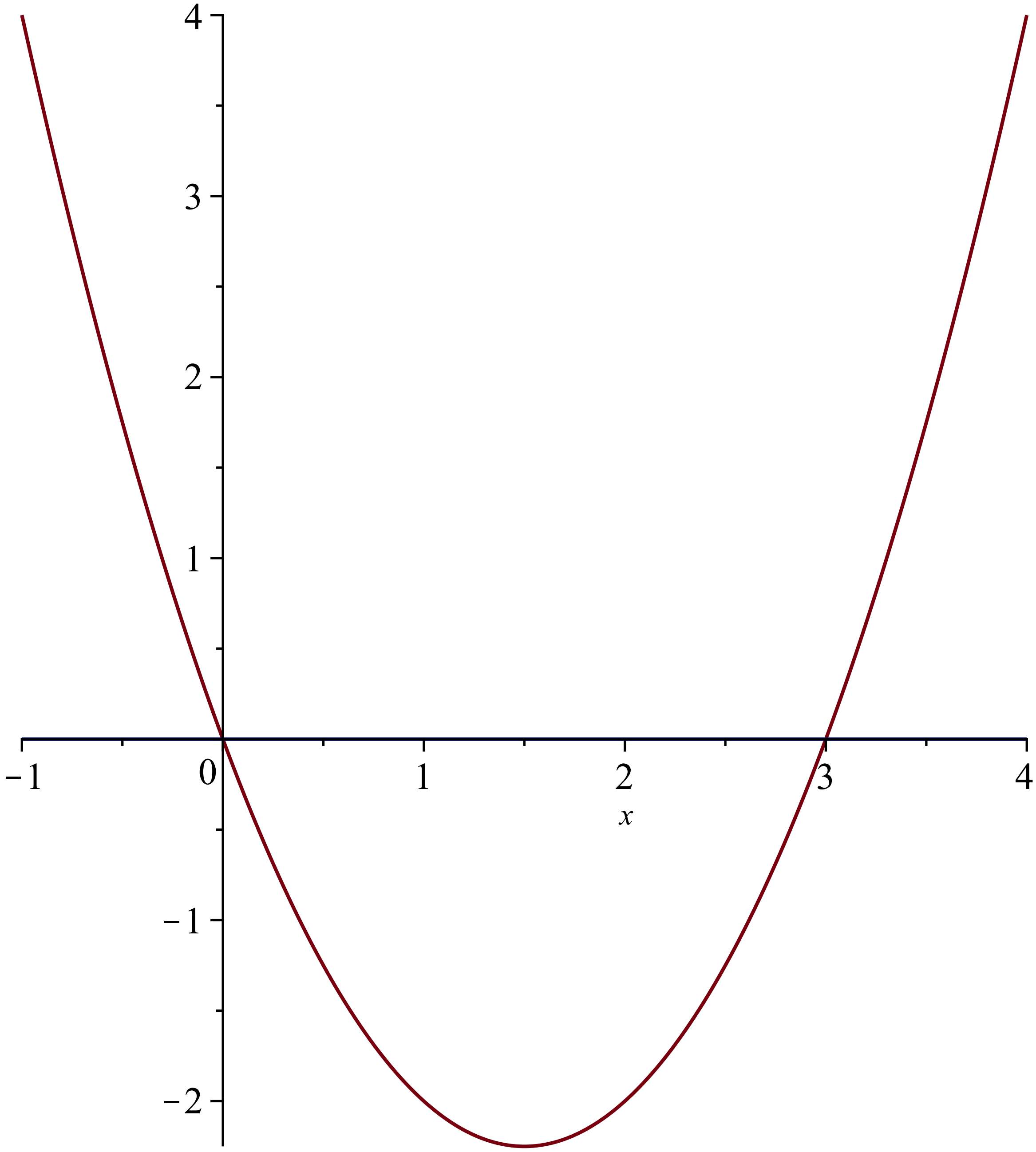
> $f1 := x^2 - 3 \cdot x - 5 < 0;$

$$f1 := x^2 - 3x < 5 \qquad (9.1)$$

> $f2 := x^{\frac{1}{2}} - 2\cdot x + 4 > 0;$

$$f2 := 0 < \sqrt{x} - 2x + 4 \quad \textbf{(9.2)}$$

> $plot([lhs(f1), lhs(f2)], x);$



> $solve([f1, f2], x);$

$$\left\{0 \le x, x < \frac{17}{8} + \frac{\sqrt{33}}{8}\right\} \quad \textbf{(9.3)}$$

> $evalf(solve([f1, f2], x));$

$$\{0. \le x, x < 2.843070331\} \quad \textbf{(9.4)}$$

s o l v e   M a p l e
:

dsolve **pdesolve** ;
isolve - ;
msolve - ;
rsolve -
;
fsolve - .

> $solve(x\verb|^|2*y+y-1);$

$$\left\{x=x,\ y=\frac{1}{x^2+1}\right\} \tag{9.5}$$

> $solve(x\verb|^|2*y+y-1, x);$ # отоносительно x

$$\frac{\sqrt{-y\,(y-1)}}{y},\ -\frac{\sqrt{-y\,(y-1)}}{y} \tag{9.6}$$

> $solve(x\verb|^|2*y+y-1, y);$ # отоносительно y

$$\frac{1}{x^2+1} \tag{9.7}$$

> $solve(\{x+2*y=5, x\verb|^|2-y\verb|^|2=8\});$

$$\{x=3,\ y=1\},\ \left\{x=-\frac{19}{3},\ y=\frac{17}{3}\right\} \tag{9.8}$$

> $solve(\{2*x+y=\mathrm{alpha}, x-y-2=0\}, \{x, y\});$

$$\left\{x=\frac{2}{3}+\frac{\alpha}{3},\ y=-\frac{4}{3}+\frac{\alpha}{3}\right\} \tag{9.9}$$

rsolve(eq,f) eq f.
f(n),
.

> $eq := 2*f(n)=3*f(n-1)-f(n-2);$

$$eq := 2f(n)=3f(n-1)-f(n-2) \tag{9.10}$$

> $rsolve(\{eq, f(1)=0, f(2)=1\}, f);$

$$2-4\left(\frac{1}{2}\right)^n \tag{9.11}$$

solve

> $F := solve(f(x)\verb|^|2-3*f(x)+2*x, f);$

$$F := x \mapsto RootOf\left(\_Z^2-3\cdot\_Z+2\cdot x\right) \tag{9.12}$$

> $f := convert(F(x), radical);$

$$f := \frac{3}{2}+\frac{\sqrt{9-8x}}{2} \tag{9.13}$$

solve _EnvExplicit:=true.

> $eq := \{\ 7*3\verb|^|x-3*2\verb|^|(z+y-x+2)=15, 2*3\verb|^|(x+1)+$

3 * 2^(z + y-x) = 66, ln(x + y + z) - 3 * ln(x) - ln(y * z) =-ln(4) };

$$eq := \{7\,3^{x} - 3\,2^{z+y-x+2} = 15,\ 2\,3^{x+1} + 3\,2^{z+y-x} = 66,\ \ln(x+y+z) - 3\ln(x) - \ln(yz) = -2\ln(2)\} \quad \textbf{(9.14)}$$

> _EnvExplicit := true :

> s := solve(eq, {x, y, z}) :

> simplify(s[1]); simplify(s[2]);

$$\{x = 2, y = 3, z = 1\}$$

$$\{x = 2, y = 1, z = 3\} \quad \textbf{(9.15)}$$