

$$7.8. 4 \cdot \sqrt[3]{81} \cdot \sqrt[6]{81} = 4 \cdot \underbrace{81^{\frac{1}{3}} \cdot 81^{\frac{1}{6}}}_{81^{\frac{1}{3} + \frac{1}{6}}} = 4 \cdot 81^{\frac{1}{2}} = 4 \cdot \sqrt{81} = 4 \cdot 9 = 36$$

$$7.9. \sqrt{28 - 16\sqrt{3}} + \sqrt{28 + 16\sqrt{3}} = \sqrt{(a-b)^2} + \sqrt{(a+b)^2} \quad \textcircled{=}$$

$$\begin{aligned} \underbrace{28}_{a^2+b^2} - \underbrace{16\sqrt{3}}_{2ab} &= (a-b)^2 = a^2 + 2 \cdot a \cdot b + b^2 \\ \begin{cases} a^2 + b^2 = 28 \\ 2ab = 16\sqrt{3} \Rightarrow ab = 8\sqrt{3} \end{cases} &\Rightarrow \begin{cases} a^2 + b^2 = 28 \\ ab = 8\sqrt{3} \end{cases} \end{aligned}$$

$\sqrt[2]{a^2} = |a|$   
 $\sqrt[4]{a^4} = |a|$   
 $\sqrt[3]{a^3} = a$

$$\begin{aligned} a &= 8 & b &= \sqrt{3} \\ \boxed{a = 4} & \quad \boxed{b = 2\sqrt{3}} & a^2 + b^2 &= 64 + 3 \neq 28 \\ & & a^2 + b^2 &= 16 + 4 \cdot 3 = 28! \end{aligned}$$

$$\textcircled{=} \sqrt{(4 - 2\sqrt{3})^2} + \sqrt{(4 + 2\sqrt{3})^2} = |4 - 2\sqrt{3}| + |4 + 2\sqrt{3}| \quad \textcircled{=}$$

$$|a| = \begin{cases} a, & a \geq 0 \\ -a, & a < 0 \end{cases} \quad \begin{aligned} a &\geq 0, a = 5 & |5| &= 5 \\ a &= 0 & |0| &= 0 \\ a &< 0, a = -3 & |-3| &= -(-3) = 3 \end{aligned}$$

$$\sqrt{3} \approx 1.7, \quad 2\sqrt{3} \approx 3.4 \Rightarrow 4 - 2\sqrt{3} > 0$$

$$\textcircled{=} 4 - 2\sqrt{3} + 4 + 2\sqrt{3} = 8$$

$$7.11. \frac{(\sqrt{33})^{n+6}}{(\sqrt{3})^{n-4}}, \text{ если } 121^{2n+10} = 3748096$$

$$\begin{aligned} \frac{(\sqrt{33})^{n+6}}{(\sqrt{3})^{n-4}} &= \frac{(\sqrt{3} \cdot \sqrt{11})^{n+6}}{(\sqrt{3})^{n-4}} = \frac{(\sqrt{3})^{n+6} \cdot (\sqrt{11})^{n+6}}{(\sqrt{3})^{n-4}} = \\ &= (\sqrt{3})^{n+6-(n-4)} \cdot (\sqrt{11})^{n+6} = (\sqrt{3})^{10} \cdot (\sqrt{11})^{n+6} \quad \textcircled{=} \end{aligned}$$

$$121^{2n+10} = 3748096 \Rightarrow 121^{2(n+5)} = 3748096 \Rightarrow (121^{n+5})^2 = 3748096 \quad \left. \begin{array}{l} \text{извлечь} \\ \text{корень} \\ \text{квадр} \end{array} \right\}$$

$$121^{n+5} = 1936 \Rightarrow (11^2)^{n+5} = 1936 \Rightarrow (11^{n+5})^2 = 1936 \quad \left. \begin{array}{l} \text{извл. } \sqrt{\phantom{x}} \end{array} \right\}$$

$$\boxed{11^{n+5} = 44} \quad \textcircled{=} \left(3^{\frac{1}{2}}\right)^{10} \cdot \left(11^{\frac{1}{2}}\right)^{n+6} = 3^5 \cdot (11^{n+6})^{\frac{1}{2}} =$$

$$= 243 \cdot (11^{n+5+1})^{\frac{1}{2}} = 243 \cdot (11^{n+5} \cdot 11)^{\frac{1}{2}} = 243 \cdot \sqrt{44 \cdot 11} =$$

$$= 243 \cdot \sqrt{4 \cdot 11^2} = 243 \cdot 2 \cdot 11 = 486 \cdot 11 = 5346$$