15.4. [Сборник ФИПИ] $\dfrac{4^{x-0,5}+1}{9\cdot 4^x-16^{x+0,5}-2}\leqslant 0,5$

$0,5=\frac{1}{2}$

$4^{x-0,5}=4^x\cdot 4^{-0,5}=4^x\cdot\frac{1}{4^{0,5}}=4^x\cdot\frac{1}{\sqrt{4}}=\frac{4^x}{2}=0,5\cdot 4^x$

$16^{x+0,5}=16^x\cdot 16^{0,5}=(4^x)^2\cdot\sqrt{16}=4\cdot(4^x)^2$

$t=4^x$

$\dfrac{0,5t+1}{9t-4t^2-2}\leq 0,5 \quad \Big|\cdot 2$

$\dfrac{2(0,5t+1)}{9t-4t^2-2}\leq 0,5\cdot 2$

$\dfrac{t+2}{9t-4t^2-2}-1\leq 0$

$\dfrac{t+2-(9t-4t^2-2)}{9t-4t^2-2}\leq 0$

$\dfrac{4t^2-8t+4}{9t-4t^2-2}\leq 0$

$\dfrac{4(t^2-2t+1)}{9t-4t^2-2}\leq 0 \quad \Big| : 4$

$\dfrac{t^2-2t+1}{9t-4t^2-2}\leq 0$

$\dfrac{(t-1)^2}{9t-4t^2-2}\leq 0$

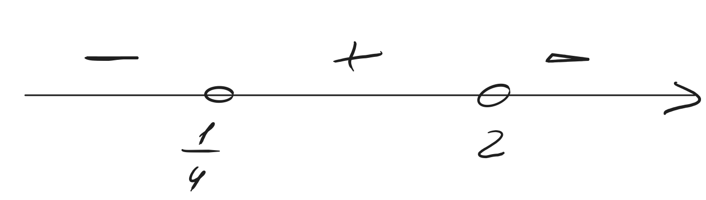
Пусть $(t-1)^2>0$. Тогда $9t-4t^2-2<0$.

$4t^2-9t+2=0$

$D=81-4\cdot4\cdot2=81-32=49$

$t_{1,2}=\dfrac{9\pm7}{8}$ $\quad t_1=\frac{2}{8}=\frac{1}{4}$

$t_2=\frac{16}{8}=2$




$\begin{cases}(t-1)^2>0 \text{ при любых } t\text{, т.е. } -\infty<t<+\infty,\ t\neq 1\\ t<\frac{1}{4},\ t>2\end{cases}$

$\Rightarrow\quad t<\frac{1}{4},\ t>2$

Если $(t-1)^2=0$, то $9t-4t^2-2\neq 0$ $\left(t\neq\frac{1}{4},\ t\neq 2\right)$

Т.О., $t<\frac{1}{4},\ t>2,\ t=1$

$\left[\begin{array}{l}4^x<\frac{1}{4}\\ 4^x>2\\ 4^x=1\end{array}\right.$ $\quad \left[\begin{array}{l}4^x<4^{-1}\\ 4^x>4^{0,5}\\ 4^x=4^0\end{array}\right.$ $\quad \left[\begin{array}{l}x<-1\\ x>0,5\\ x=0\end{array}\right.$

Ответ: $(-\infty;-1)\cup\{0\}\cup(0,5;+\infty)$