Логарифм

Определение $\log_a x = y \iff a^y = x$

a – основание логарифма; $a>0,\ a \neq 1$

x – аргумент логарифма; $x>0$

Пример: $\log_2 64 = y?$ $2^y = 64 \Rightarrow y=6$

$4^x = 3 \Rightarrow x = \log_4 3$

Свойства

$\ln x$ – натуральный; $\log_e x = \ln x$

$e \approx 2{,}71828$

$\lg x$ – десятичный; $\log_{10} x = \lg x$

1. $\log_a 1 = 0$
2. $\log_a a = 1$
3. $\log_a x^p = p \cdot \log_a x$
4. $\log_{a^p} x = \frac{1}{p} \log_a x$
5. $\log_a x + \log_a y = \log_a (xy)$
6. $\log_a x - \log_a y = \log_a \left(\frac{x}{y}\right)$
7. $a^{\log_a x} = x$; $a^{\log_a b} = b^{\log_a a}$
8. формула перехода к новому основанию

$\log_a x = \frac{\log_b x}{\log_b a}$

частный случай: $\log_a x = \frac{1}{\log_x a}$

$\left(\log_a x = \frac{\log_x x}{\log_x a} = \frac{1}{\log_x a}\right)$

$\left(\text{св-во 4: } \log_{a^p} x = \frac{\log_a x}{\log_a a^p} = \frac{\log_a x}{p \log_a a} = \frac{1}{p}\log_a x\right)$

Уравнения

1) $\log_a f(x) = \log_a g(x) \iff f(x) = g(x)$

$a>0, a\neq 1$; $f(x)>0$, $g(x)>0$

$\log_{a(x)} f(x) = \log_{a(x)} g(x) \iff \begin{cases} f(x) = g(x) \\ a(x) > 0 \\ a(x) \neq 0 \end{cases}$

2) $\log_a f(x) = b$ по определению

$f(x) = a^b$

6.1. $\log_4(2x-7) = 3$

6.2. $\log_{0,5}(5{,}5-5x) = -5$

6.3. $\log_x 81 = 4$

6.4. $\log_{3-2x} 6{,}25 = 2$

6.5. $\log_4(x+3) = \log_4(4x-15)$

6.6. $\log_9(x^2-9x) = \log_9(72-8x)$

6.7. $\log_5(7-x) = \log_5(3-x)+1$

$\boxed{6.1}$ $\log_4(2x-7) = 3$

$2x-7 = 4^3 = 64$

$2x = 71 \Rightarrow x = 35{,}5$

$\boxed{6.3}$ $\log_x 81 = 4$

$x^4 = 81 = 3^4$

$x = 3$

$\boxed{6.5}$ $\log_4(x+3) = \log_4(4x-15)$

$x+3 = 4x-15$

$-3x = -18$

$x = 6$

$\boxed{6.7}$ I способ $\log_5 5 = 1$

$\log_5(7-x) = \log_5(3-x) + \log_5 5$

$\log_5(7-x) = \log_5(5(3-x))$

$7-x = 15-5x$

$4x = 8$

$x = 2$

II способ

$\log_5(7-x) - \log_5(3-x) = 1$

$\log_5 \frac{7-x}{3-x} = 1 \Rightarrow \frac{7-x}{3-x} = 5^1 \Rightarrow \frac{7-x}{3-x} - 5 = 0$

$\frac{7-x-5(3-x)}{3-x} = 0$

$3-x \neq 0$

$7-x-5(3-x) = 0$

$x = 2$

6.8. $\log_2\left(\frac{1}{8}-x\right) = \log_2\left(3x+\frac{1}{8}\right) - 1$

6.9. $\log_3(4x-7) = 3\log_3 6$

6.10. $\log_5(5-x) = 2\log_5 3$

6.11. $\log_8 2^{8x-4} = 4$

6.12. $\log_{27} 3^{5x+5} = 2$

6.13. $2^{\log_8(5x-3)} = 4$

6.14. $3^{\log_9(2x+7)} = 2$

$\boxed{6.9}$ $\log_3(4x-7) = \underline{3}\log_3 6$

$\log_3(4x-7) = \log_3 6^3$

$4x-7 = 6^3 = 216$

$4x = 223$

$x = 55{,}75$

$\boxed{6.11}$ $\log_8 2^{8x-4} = 4$

$2^{8x-4} = 8^4 = (2^3)^4$

$2^{8x-4} = 2^{12} \Rightarrow 8x-4 = 12$

$x = 2$

$\boxed{6.13}$ $2^{\log_8(5x-3)} = 4$

$2^{\log_8(5x-3)} = 2^2$

$\log_8(5x-3) = 2$

$5x-3 = 8^2 = 64$

$5x = 67$

$x = 13{,}4$

6.2	−5,3	6.8	0,025
6.4	0,25	6.10	−4
·6.6	−8	6.12	0,2
		6.14	−1,5

$\boxed{6.6}$ $\cancel{x_1 = 9}$, $x_2 = -8$

$\begin{cases} x^2 - 9x > 0 \\ 72-8x > 0 \end{cases}$

$\boxed{6.8}$ $\log_2\left(\frac{1}{8}-x\right) = \log_2\left(3x+\frac{1}{8}\right) - 1$

$\log_2\left(\frac{1}{8}-x\right) = \log_2\left(3x+\frac{1}{8}\right) - \log_2 2$

$\log_2\left(\frac{1}{8}-x\right) = \log_2\left(\frac{1}{2}\left(3x+\frac{1}{8}\right)\right)$

$\frac{1}{8} - x = \frac{1}{2}\left(3x+\frac{1}{8}\right) \quad | \cdot 2$

$\frac{2}{8} - 2x = 3x + \frac{1}{8}$

$-5x = -\frac{1}{8}$

$x = \frac{1}{40} = 0{,}025$