6.15. $\lg^2(10x) - \lg(1000x) = 4$

$\lg^2(10x) = (\lg(10x))^2 = (\lg 10 + \lg x)^2 = (1 + \lg x)^2$

$\lg(1000x) = \lg 1000 + \lg x = 3 + \lg x$

$\lg 1000 = y \Rightarrow 10^y = 1000 \Rightarrow y = 3$

$\lg 1000 = \lg 10^3 = 3 \cdot \lg 10 = 3 \cdot 1 = 3$

$(1 + \lg x)^2 - (3 + \lg x) - 4 = 0$

Замена $t = \lg x \quad (x > 0)$

$(1 + t)^2 - 3 - t - 4 = 0$

$1 + 2t + t^2 - t - 7 = 0$

$t^2 + t - 6 = 0$

$t_1 + t_2 = -1$

$t_1 \cdot t_2 = -6$

$t_1 = -3, \quad t_2 = 2$

$\lg x = -3$

$x_1 = 10^{-3}$

$x_1 = 0{,}001$

$\lg x = 2$

$x_2 = 10^2$

$x_2 = 100$

$t = a^x > 0$

$\log_3 \frac{1}{9} = -2$

$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

6.14. $3^{\log_9(2x+7)} = 2$

$3^{\log_3 f(x)} = f(x)$

$\log_9(2x+7) = \log_{3^2}(2x+7) = \frac{1}{2}\log_3(2x+7) =$

$= \log_3 \sqrt{2x+7}$

$3^{\log_3 \sqrt{2x+7}} = 2 \Rightarrow \sqrt{2x+7} = 2$

$2x + 7 = 4 \Rightarrow 2x = -3; \quad x = -1{,}5$

II способ

$(x^a)^b = x^{ab} = x^{ba} = (x^b)^a$

$\left(3^{\log_9(2x+7)}\right)^2 = 2^2$

$\left(3^2\right)^{\log_9(2x+7)} = 4$

$9^{\log_9(2x+7)} = 4$

$2x + 7 = 4 \Rightarrow x = -1{,}5$

6.16. $\log_5^2(125x) - \log_5(5x) = 14$

$\log_5^2(125x) = (3 + \log_5 x)^2$

$\log_5(5x) = \log_5 5 + \log_5 x = 1 + \log_5 x$

Замена: $t = \log_5 x$

$(3 + t)^2 - (1 + t) = 14$

$t^2 + 5t - 6 = 0$

$t_1 = -6; \quad t_2 = 1$

$\log_5 x = -6$

$x_1 = 5^{-6}$

$x_1 = \frac{1}{5^6} = \frac{1}{(5^3)^2}$

$x_1 = \frac{1}{125^2} = \frac{1}{15625}$

$125^2 = 15625$

$125 \cdot 13 = 156$

$\log_5 x = 1$

$x_2 = 5^1$

$x_2 = 5$