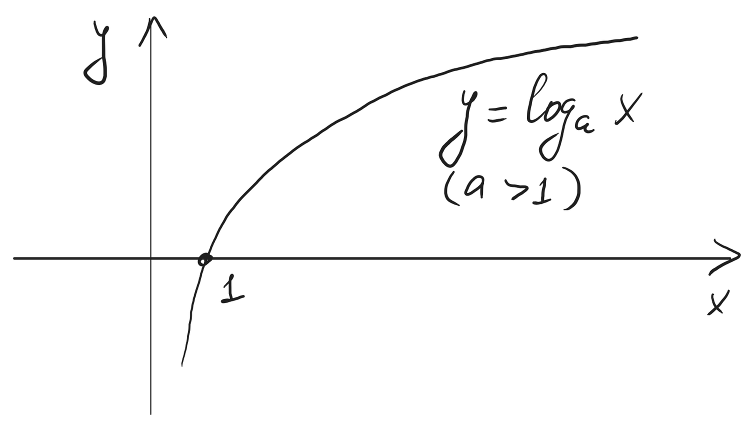
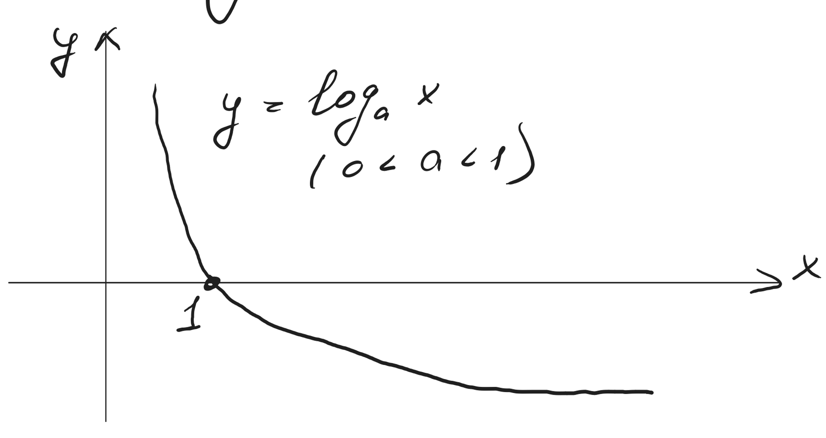
Логарифмические нер-ва

$\log_a f(x) < \log_a g(x)$

1) если $0 < a < 1$, то $f(x) > g(x)$

2) если $a > 1$, то $f(x) < g(x)$





► 1. $\log_{11}(3x-1) > 1$

► 2. $\log_{\frac{1}{3}}(7x-1) > 0$

► 3. $\lg(x^2+5x+7) < 0$

► 4. $\log_{0,5}(x^2+5x+6) > -1$

(1) $\log_{11}(3x-1) > \log_{11} 11$

т.к. $11 > 1$, то

$3x - 1 > 11$

$3x > 12$

$x > 4$

ОДЗ
$3x - 1 > 0$
$x > \frac{1}{3}$

Ответ: $x > 4$

(2) $\log_{\frac{1}{3}}(7x-1) > \log_{\frac{1}{3}} 1$

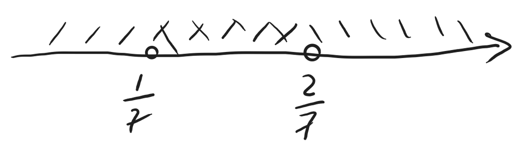
т.к. $0 < \frac{1}{3} < 1$, то

$7x - 1 < 1$

$7x < 2$

$x < \frac{2}{7}$

ОДЗ
$7x - 1 > 0$
$x > \frac{1}{7}$



Ответ: $\frac{1}{7} < x < \frac{2}{7}$

(3) $\lg(x^2+5x+7) < \lg 1$

т.к. $10 > 1$, то

$x^2 + 5x + 7 < 1$

$x^2 + 5x + 6 < 0$

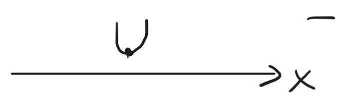
$x^2 + 5x + 6 = 0$

$x_1 + x_2 = -5$

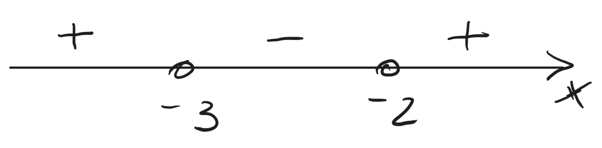
$x_1 \cdot x_2 = +6$

$x_1 = -3, \quad x_2 = -2$

ОДЗ
$x^2 + 5x + 7 > 0$
$D = 25 - 4 \cdot 7 < 0$



$-\infty < x < \infty$



$-3 < x < -2$ — ответ

(4) $\log_{0,5}(x^2+5x+6) > \log_{0,5} 2$

т.к. $0 < 0,5 < 1$, то

$x^2 + 5x + 6 < 2$

$x^2 + 5x + 4 < 0$

$x^2 + 5x + 4 = 0$

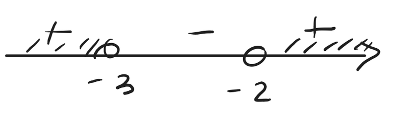
$x_1 + x_2 = -5$

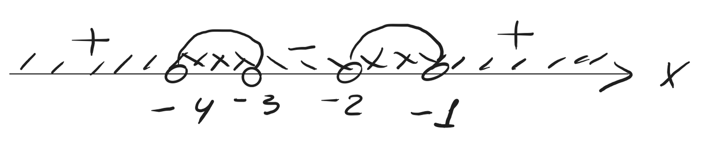
$x_1 \cdot x_2 = 4$

$x_1 = -1, \quad x_2 = -4$

$-4 < x < -1$

ОДЗ
$x^2 + 5x + 6 > 0$
$x^2 + 5x + 6 = 0$
$x_1 = -2, \ x_2 = -3$





Ответ: $(-4; -3) \cup (-2; -1)$

► 5. $\log_{\sqrt{2}}\left(\frac{1-2x}{x}\right) \leqslant 0$

► 6. $\log_{\frac{1}{5}}(3x-4) > \log_{\frac{1}{5}}(x-2)$

► 7. $\log_{0,1}(4-x) \geqslant \log_{0,1} 10 - \log_{0,1}(x-1)$

(5) $\log_{\sqrt{2}}\left(\frac{1-2x}{x}\right) \leq \log_{\sqrt{2}} 1$

$\sqrt{2} > 1 \Rightarrow$

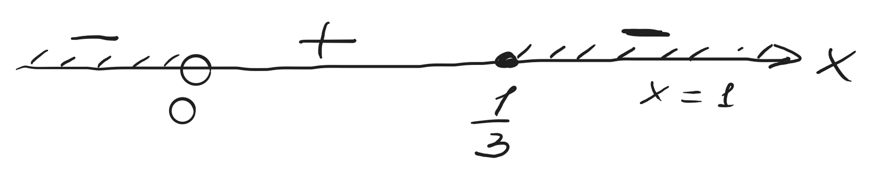
$\frac{1-2x}{x} \leq 1 \Rightarrow \frac{1-2x}{x} - 1 \leq 0$

$\frac{1-2x-x}{x} \leq 0 \Rightarrow \frac{1-3x}{x} \leq 0$

$1 - 3x = 0$

$x = \frac{1}{3}$

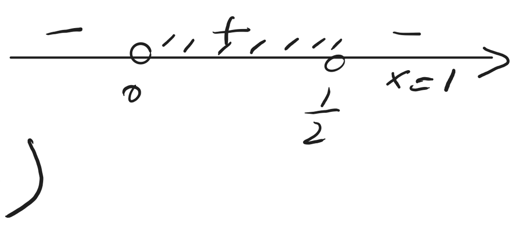
$x \neq 0$



$\left[\begin{array}{l} x < 0 \\ x \geqslant \frac{1}{3} \end{array}\right.$

ОДЗ
$\frac{1-2x}{x} > 0$

$1 - 2x = 0$
$x \neq 0$



Ответ: $[\frac{1}{3}; \frac{1}{2})$

(6) $0 < \frac{1}{5} < 1 \Rightarrow$

$3x - 4 < x - 2$

$3x - x < -2 + 4$

$2x < 2$

$x < 1$

ОДЗ
$\begin{cases} 3x - 4 > 0 \\ x - 2 > 0 \end{cases}$ $\begin{cases} x > \frac{4}{3} \\ x > 2 \end{cases}$ $x > 2$

Ответ: $\varnothing$

(7) $\log_{0,1}(4-x) + \log_{0,1}(x-1) \geqslant \log_{0,1} 10$

$\log_{0,1}(4-x)(x-1) \geqslant \log_{0,1} 10$

$(4-x)(x-1) \leq 10$

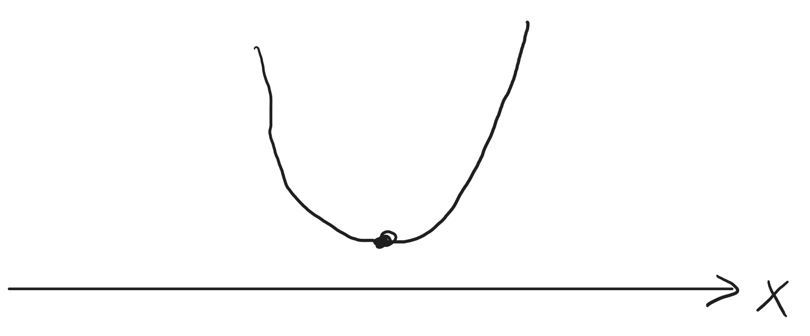
$4x - 4 - x^2 + x - 10 \leq 0$

$-x^2 + 5x - 14 \leq 0$

$x^2 - 5x + 14 \geqslant 0$

$x^2 - 5x + 14 = 0$

$D = 25 - 4 \cdot 14 = 25 - 56 < 0$



любые x

$\begin{cases} -\infty < x < +\infty \\ 4 - x > 0 \\ x - 1 > 0 \end{cases}$ $\begin{cases} -\infty < x < +\infty \\ x < 4 \\ x > 1 \end{cases} \Rightarrow$ $1 < x < 4$ — ответ

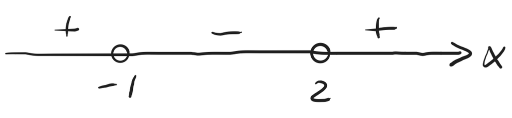
► 15. $\log_{0,1}(x^2 - x - 2) > \log_{0,1}(3 - x)$

$\begin{cases} x^2 - x - 2 > 0 \\ 3 - x > 0 \\ x^2 - x - 2 < 3 - x \end{cases}$ $\begin{cases} x < -1, \ x > 2 \\ x < 3 \\ -\sqrt{5} < x < \sqrt{5} \end{cases}$

1) $x^2 - x - 2 > 0$

$x^2 - x - 2 = 0$

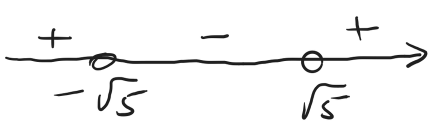
$x_1 = -1; \ x_2 = 2$

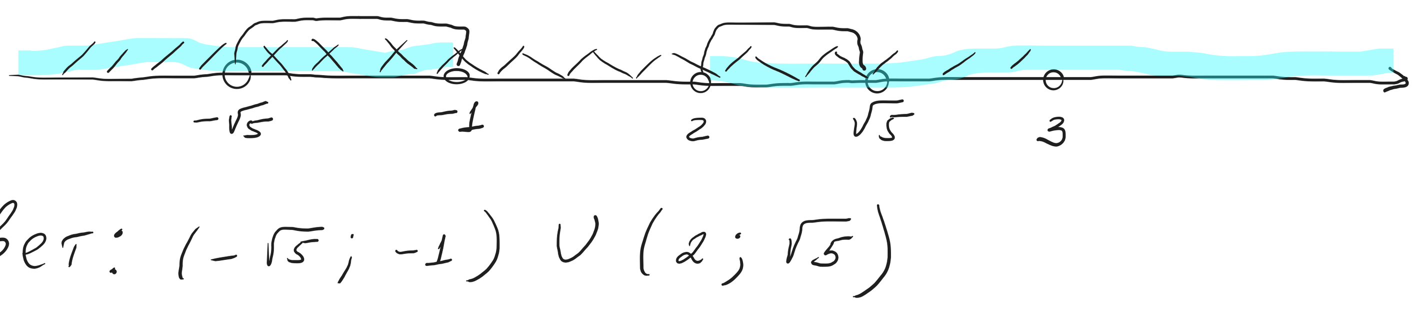


3) $x^2 - 5 < 0$

$x^2 - 5 = 0$

$x_{1,2} = \pm\sqrt{5}$





Ответ: $(-\sqrt{5}; -1) \cup (2; \sqrt{5})$