6.9. (★) $2\sin^2\frac{\pi x}{12} + 3\cos\frac{\pi x}{12} = 0$

1) преобр. аргумента
2) преобр. ф-ции (одн.)

$\sin^2\alpha + \cos^2\alpha = 1$
$\sin^2\alpha = 1 - \cos^2\alpha$

$2\sin^2\frac{\pi x}{12} + 3\cos\frac{\pi x}{12} = 0$

$2\left(1-\cos^2\frac{\pi x}{12}\right) + 3\cos\frac{\pi x}{12} = 0$

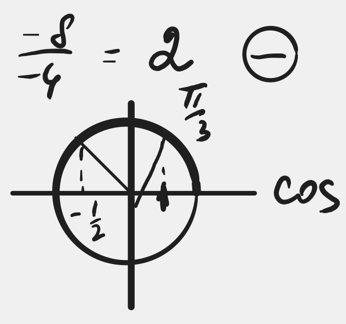
$t = \cos\frac{\pi x}{12}, \quad |t| \le 1$

$2(1-t^2) + 3t = 0 \quad \Rightarrow \quad -2t^2 + 3t + 2 = 0$

$D = 9 + 16 = 25 \quad \Rightarrow \quad t_{1,2} = \frac{-3 \pm 5}{-4}$

$t_1 = \frac{-8}{-4} = 2$ ⊖

$t_2 = \frac{2}{-4} = -\frac{1}{2}$



$\cos\frac{\pi x}{12} = -\frac{1}{2}$

$\frac{\pi x}{12} = \pm\arccos\left(-\frac{1}{2}\right) + 2\pi n, \quad n \in \mathbb{Z}$

$\arccos\left(-\frac{1}{2}\right) = \pi - \arccos\frac{1}{2} = \pi - \frac{\pi}{3} = \frac{2}{3}\pi$

$\frac{\pi x}{12} = \pm\frac{2}{3}\pi + 2\pi n \quad \Big| \cdot \frac{12}{\pi}$

$x = \pm 8 + 24n, \quad n \in \mathbb{Z}$

6.11. (★) $10\,\mathrm{tg}\,\overset{\alpha}{\boxed{\pi x}} + \mathrm{ctg}\,\overset{\alpha}{\boxed{\pi x}} = 11$

$\mathrm{tg} = \frac{\sin}{\cos}$; $\mathrm{ctg} = \frac{\cos}{\sin}$ $\Big| \Rightarrow$ $\mathrm{tg} = \frac{1}{\mathrm{ctg}}$; $\mathrm{ctg} = \frac{1}{\mathrm{tg}}$

1) $10\,\mathrm{tg}\,\alpha + \frac{1}{\mathrm{tg}\,\alpha} = 11 \quad \Rightarrow \quad \frac{10\,\mathrm{tg}^2\alpha + 1}{\mathrm{tg}\,\alpha} = 11$

$t = \mathrm{tg}\,\alpha \ne 0$

$\frac{10t^2+1}{t} = 11$; $10t^2 - 11t + 1 = 0$

$D = 121 - 40 = 81 = 9^2$

$t_{1,2} = \frac{11 \pm 9}{20} \quad \Rightarrow \quad t_1 = 1, \; t_2 = 0{,}1$

$\mathrm{tg}\,x = a \quad \left(x \ne \frac{\pi}{2} + \pi n\right) \quad \Rightarrow \quad x = \mathrm{arctg}\,a + \pi n, \quad n \in \mathbb{Z}$

$\mathrm{tg}\,\pi x = 1$

$\pi x = \frac{\pi}{4} + \pi n, \; n \in \mathbb{Z}$

$x = \frac{1}{4} + n, \; n \in \mathbb{Z}$

$\mathrm{tg}\,\pi x = 0{,}1$

$\pi x = \mathrm{arctg}\,0{,}1 + \pi k, \; k \in \mathbb{Z}$

$x = \frac{1}{\pi}\,\mathrm{arctg}\,0{,}1 + k, \; k \in \mathbb{Z}$

2) $10\frac{\sin\alpha}{\cos\alpha} + \frac{\cos\alpha}{\sin\alpha} = 11 \Big| \begin{matrix}\cdot\sin\alpha \\ \cdot\cos\alpha\end{matrix}$

$\cos\alpha \ne 0$
$\sin\alpha \ne 0$

$10\sin^2\alpha + \cos^2\alpha = 11\sin\alpha\cos\alpha$

$10\sin^2\alpha + \cos^2\alpha - 11\sin\alpha\cos\alpha = 0$

$10\sin^2\alpha + \cos^2\alpha - 10\sin\alpha\cos\alpha - \sin\alpha\cos\alpha = 0$

$10\sin\alpha(\sin\alpha - \cos\alpha) + \cos\alpha(\cos\alpha - \sin\alpha) = 0$

$(\sin\alpha - \cos\alpha)(10\sin\alpha - \cos\alpha) = 0$

$\sin\alpha - \cos\alpha = 0$

$\sin\alpha = \cos\alpha \;|:\cos\alpha$

$\mathrm{tg}\,\alpha = 1$

$10\sin\alpha - \cos\alpha = 0$

$10\sin\alpha = \cos\alpha \;|:\cos\alpha : 10$

$\mathrm{tg}\,\alpha = \frac{1}{10}$

6.13. (★) $2\sin^2 x + 3\sin x\cos x + \cos^2 x = 0$

$2\sin^2 x + 3\sin x\cos x + \cos^2 x = 0$

$\underline{2\sin^2 x + 2\sin x\cos x} + \underline{\sin x\cos x + \cos^2 x} = 0$

$2\sin x(\sin x + \cos x) + \cos x(\sin x + \cos x) = 0$

$(\sin x + \cos x)(2\sin x + \cos x) = 0$

$\sin x + \cos x = 0 \;|:\cos x$

$\mathrm{tg}\,x + 1 = 0$

$\mathrm{tg}\,x = -1$

$x = -\frac{\pi}{4} + \pi k, \; k \in \mathbb{Z}$

$2\sin x + \cos x = 0 \;|:\cos x$

$2\,\mathrm{tg}\,x + 1 = 0$

$\mathrm{tg}\,x = -\frac{1}{2}$

$x = -\mathrm{arctg}\,\frac{1}{2} + \pi n, \; n \in \mathbb{Z}$