6.9. (★) $2\sin^2\frac{\pi x}{12} + 3\cos\frac{\pi x}{12} = 0$

$$2\cdot\left(1 - \cos^2\frac{\pi x}{12}\right) + 3\cos\frac{\pi x}{12} = 0$$

$$-2\cos^2\frac{\pi x}{12} + 3\cos\frac{\pi x}{12} + 2 = 0$$

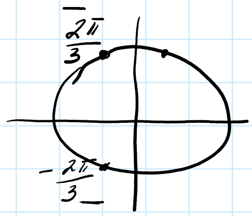
Замена: $t = \cos\frac{\pi x}{12}$; $-1 \le t \le 1$

$$-2t^2 + 3t + 2 = 0$$

$$D = 3^2 - 4\cdot(-2)\cdot 2 = 9 + 16 = 25 = 5^2$$

$t_{1,2} = \frac{-3 \pm 5}{2\cdot(-2)}$; $t_1 = \frac{-3-5}{-4} = \frac{-8}{-4} = 2$ не удовл. ОДЗ

$t_2 = \frac{-3+5}{-4} = \frac{2}{-4} = -\frac{1}{2}$



$\cos\frac{\pi x}{12} = -\frac{1}{2} \Rightarrow \frac{\pi x}{12} = \pm\frac{2\pi}{3} + 2\pi k,\ k\in\mathbb{Z}$

$x = \pm 8 + 24k,\ k\in\mathbb{Z}$

Ответ: $\pm 8 + 24k,\ k\in\mathbb{Z}$

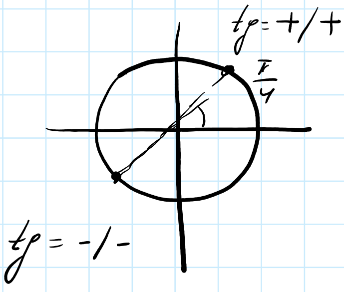
$tg\,\alpha = \frac{1}{ctg\,\alpha}$; $ctg\,\alpha = \frac{1}{tg\,\alpha}$

6.11. (★) $10\,\mathrm{tg}\,\pi x + \mathrm{ctg}\,\pi x = 11$

Замена: $t = tg\,\pi x \Rightarrow ctg\,\pi x = \frac{1}{t}$

$10t + \frac{1}{t} = 11 \Rightarrow \frac{10t^2 - 11t + 1}{t} = 0$

$$\begin{cases} 10t^2 - 11t + 1 = 0 \\ t \ne 0 \end{cases}$$



$$D = (-11)^2 - 4\cdot 10\cdot 1 = 121 - 40 = 81 = 9^2$$

$t_{1,2} = \frac{11 \pm 9}{2\cdot 10} \Rightarrow \left[\begin{array}{l} t_1 = 1 \\ t_2 = \frac{1}{10} \end{array}\right.$

$tg\,\pi x = 1$

$\pi x = \frac{\pi}{4} + \pi k,\ k\in\mathbb{Z}$

$x = \frac{1}{4} + k,\ k\in\mathbb{Z}$

$tg\,\pi x = \frac{1}{10}$

$\pi x = arctg\frac{1}{10} + \pi n,\ n\in\mathbb{Z}$

$x = \frac{1}{\pi}arctg\frac{1}{10} + n,\ n\in\mathbb{Z}$

Ответ: $\frac{1}{4}+k,\ k\in\mathbb{Z}$; $\frac{1}{\pi}\operatorname{arctg}\frac{1}{10}+n,\ n\in\mathbb{Z}$

6.13. (★) $2\sin^2 x + 3\sin x\cos x + \cos^2 x = 0$

$a\sin^2 x + b\sin x\cos x + c\cos^2 x = 0 \ \Big|\ \begin{array}{l} :\sin^2 x \\ :\cos^2 x \end{array}$

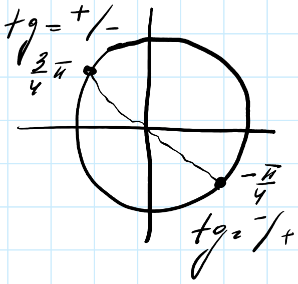
$\left[\begin{array}{l} a\,tg^2 x + b\,tg\,x + c = 0 \\ \cos^2 x = 0\ ? \end{array}\right.$

$2tg^2 x + 3tg\,x + 1 = 0$

Замена: $t = tg\,x$

$2t^2 + 3t + 1 = 0$; $D = 3^2 - 4\cdot 2\cdot 1 = 9 - 8 = 1$

$t_{1,2} = \frac{-3\pm 1}{2\cdot 2}$; $\left[\begin{array}{l} t_1 = -1 \\ t_2 = -\frac{1}{2} \end{array}\right.$



$tg\,x = -1$

$x = -\frac{\pi}{4} + \pi k,\ k\in\mathbb{Z}$

$tg\,x = -\frac{1}{2}$

$x = \operatorname{arctg}\left(-\frac{1}{2}\right) + \pi n,\ n\in\mathbb{Z}$

$x = \pi - \operatorname{arctg}\frac{1}{2} + \pi n,\ n\in\mathbb{Z}$

(*) Проверить: $\cos^2 x = 0 \Rightarrow \cos x = 0 \Rightarrow \sin^2 x = 1 \Rightarrow \sin x = \pm 1$

1) $2\cos^2 x + 3\sin x\cos x + \sin^2 x = 0$ при $\cos x = 0$, $\sin x = 1$

$2\cdot 0^2 + 3\cdot 1\cdot 0 + 1^2 = 0$ — неверно

2) $2\cdot\cos^2 x + 3\sin x\cos x + \sin^2 x = 0$ при $\cos x = 0$, $\sin x = -1$

$2\cdot 0^2 + 3\cdot(-1)\cdot 0 + (-1)^2 = 0$ — неверно

Вместо (*) можно написать: "Очевидно, что при делении на $\cos^2 x$ не были потеряны решения"

Ответ: $-\frac{\pi}{4} + \pi k,\ k\in\mathbb{Z}$; $\pi - \operatorname{arctg}\frac{1}{2} + \pi n,\ n\in\mathbb{Z}$