13.1. [демо-2023] а) Решить уравнение

$$2\sin\left(x+\frac{\pi}{3}\right)+\cos 2x=\sqrt{3}\cos x+1$$

б) Укажите корни этого уравнения, принадлежащие отрезку $\left[-3\pi;-\frac{3\pi}{2}\right]$.

Решение:

а) $2\sin\left(x+\frac{\pi}{3}\right)+\cos 2x=\sqrt{3}\cos x+1$

$2\left(\sin x\cos\frac{\pi}{3}+\cos x\sin\frac{\pi}{3}\right)+\cos^2 x-\sin^2 x-\sqrt{3}\cos x-1=0$

$\sin x+\cancel{\sqrt{3}\cos x}+\cos^2 x-\sin^2 x-\cancel{\sqrt{3}\cos x}-1=0$

$\sin x+\underbrace{\cancel{1}-\sin^2 x}_{\cos^2 x}-\sin^2 x-\cancel{1}=0$

$-2\sin^2 x+\sin x=0$

$\sin x\,(-2\sin x+1)=0$

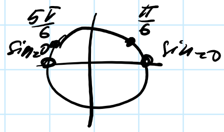
$\sin x=0$

$x=\pi k,\ k\in\mathbb{Z}$

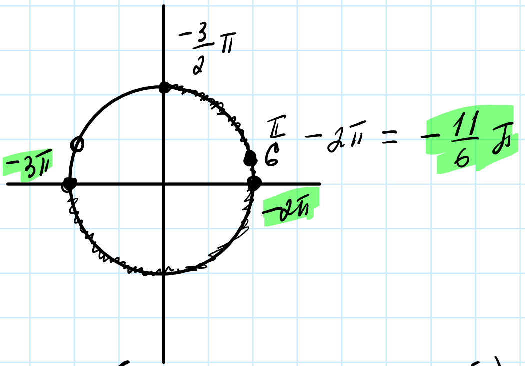
$-2\sin x+1=0$

$\sin x=\frac{1}{2}$

$$\left[\begin{array}{l} x=\frac{\pi}{6}+2\pi n,\ n\in\mathbb{Z} \\ x=\frac{5\pi}{6}+2\pi m,\ m\in\mathbb{Z} \end{array}\right.$$



б)



2 способ (аналитический)

1) $-3\pi\le\pi k\le-\frac{3}{2}\pi,\quad k\in\mathbb{Z}$

$-3\le k\le-\frac{3}{2},\ k\in\mathbb{Z}\ \Rightarrow\ k=-3,-2$

$x=-3\pi,\quad x=-2\pi$

2) $-3\pi\le\frac{\pi}{6}+2\pi n\le-\frac{3}{2}\pi,\quad n\in\mathbb{Z}$

$$-3 \le \frac{1}{6} + 2n \le -\frac{3}{2}, \quad n \in \mathbb{Z}$$

$$-\frac{19}{6} \le 2n \le -\frac{5}{3}, \quad n \in \mathbb{Z}$$

$$-\frac{19}{12} \le n \le -\frac{5}{6}, \quad n \in \mathbb{Z} \quad \Rightarrow \quad n = -1$$

$$x = \frac{\pi}{6} - 2\pi = -\frac{11}{6}\pi$$

3) $$-3\pi \le \frac{5\pi}{6} + 2\pi m \le -\frac{3}{2}\pi, \quad m \in \mathbb{Z}$$

$$-3 \le \frac{5}{6} + 2m \le -\frac{3}{2}, \quad m \in \mathbb{Z}$$

$$-\frac{23}{6} \le 2m \le -\frac{7}{3}, \quad m \in \mathbb{Z}$$

$$-\frac{23}{12} \le m \le -\frac{7}{6}, \quad m \in \mathbb{Z} \quad \Rightarrow \quad \varnothing$$

Ответ: а) $\pi k, k \in \mathbb{Z}$; $\frac{\pi}{6} + 2\pi n, n \in \mathbb{Z}$; $\frac{5\pi}{6} + 2\pi m, m \in \mathbb{Z}$

б) -3π; -2π; $-\frac{11}{6}\pi$