Practical assignment 5. Eigenvalues and their multiplicities, matrix factorizations, Hermitian and positive definite matrices

1) **Multiplicities of eigenvalues.** For the following matrices compute eigenvalues and eigenvectors, calculate algebraic and geometric multiplicity for each eigenvalue. Determine whether each eigenvalue is simple, multiple, semisimple, defective or derogatory. Is the matrix simple or semisimple? If the matrix is semisimple, try to find its diagonal form using similarity transformation.

$$A = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} \qquad\qquad A = \begin{bmatrix} 1 & 0 & 2 \\ -1 & 1 & 3 \\ 0 & 0 & 2 \end{bmatrix}$$

2) **Matlab commands for matrix factorizations and eigenvalues.** Generate a random complex square matrix and a random real square matrix of size 5. Find their eigenvalues and singular values using `eig` and `svds` commands. Compute QR, Schur, SVD and LU factorizations using Matlab commands `qr`, `schur`, `svd`, `lu`. Study help files for these commands and explore different calls of these functions, such as `[Q,R]=qr(A)`, `[U,T]=schur(A)`, `[U,S,V]=svd(A)`, `[L,U]=lu(A)`, `[L,U,P] = lu(A)`. <u>Check that each factorization gives you initial matrix A</u>. Answer the following questions:
   - How to compute full and reduced (thin) QR-factorization in Matlab? Is there any difference for full and reduced QR-factorization for a square matrix?
   - How to compute real and complex Schur form in Matlab?
   - What is the difference between LU- and PLU-factorization? How matrices L differe in each case?
   - Which factorizations contain as their entries: 1) eigenvalues of the matrix A, 2) singular values of the matrix A?

3) **Hermitian and positive definite matrices.** Generate a random complex square matrix A of size n. Then generate it's complex conjugate transpose A^H using either `A'` or `ctranspose(A)` command.
   - Compute $B = (A + A^H)/2$ and check that it will be Hermitian. Possible ways: use `ishermitian(B)` command (available in higher versions of Matlab), `isequal(B, B')` or compare maximum column (or row) sum norms `norm(B-B',1)`.
   - Compute AA^H and $A^H A$. Check whether or not these matrices are Hermitian for any matrix A.
   - Check whether or not the matrices AA^H and $A^H A$ are Hermitian positive definite for any matrix A. Positive definiteness can be checked by performing the Cholesky factorization (`chol` command in Matlab). Try to make one row or column of A zero and see the result.
   - Check whether the matrix $AA^H + I$ is Hermitian positive definite for any matrix A.