Преоброзование рациональных выражений

$$7\frac{7}{13} : \frac{7}{13} = \frac{13\cdot 7+7}{13} : \frac{7}{13} =$$

$$= \frac{98}{\cancel{13}} \cdot \frac{\cancel{13}}{7} = \frac{98}{7} = \boxed{14}$$

$$\left(5\frac{3}{4} - 10{,}2\right)\cdot\frac{2}{89} = \left(\frac{23^{(5}}{4} - \frac{102^{(2}}{10}\right)\cdot\frac{2}{89} =$$

$$= \frac{115-204}{\cancel{20}\ 10}\cdot\frac{\cancel{2}}{89} = -\frac{89}{10}\cdot\frac{1}{\cancel{89}} = -\frac{1}{10} = \boxed{-0{,}1}$$

$$\frac{0{,}9^{(10}}{6}\cdot\frac{4{,}25^{(100}}{5{,}1} = \frac{\cancel{9}\ 3}{2\,\cancel{6}\cdot\cancel{10}\ 2}\cdot\frac{\cancel{425}\ \cancel{85}\ 17}{\cancel{51}_{17}\cdot\cancel{10}\ 2} = \frac{\cancel{17}}{2\cdot 2\cdot\cancel{17}\cdot 2} =$$

$$= \frac{1}{8} = \boxed{0{,}125}$$

число^2, оканч. на 5

$\underbrace{X}_{}5^2 = X\cdot(X+1)$ допис. 25

$\underbrace{1}_{}5^2 = 1\cdot 2$ доп. 25 $= 225$

$\underbrace{4}_{}5^2 = 4\cdot 5$ доп 25 $= 2025$

$\underbrace{11}_{}5^2 = 11\cdot 12$ доп 25 $= 13225$

Возвед. в квадрат с помощью формула квадрат разности и квадрат суммы

$$76^2 = (80-4)^2 = 80^2 - 2\cdot 80\cdot 4 + 4^2 =$$
$$= 6400 - 640 + 16 = 5760 + 16 = 5776$$

$$76^2 = (75+1)^2 = 75^2 + 2\cdot 75\cdot 1 + 1^2 =$$
$$= 5625 + 150 + 1 = 5776$$

$$(413^2 - 63^2) : 8330 = \frac{413^2 - 63^2}{8330} =$$
$$= \frac{(413-63)\cdot(413+63)}{8330} = \frac{35\cancel{0}\cdot 476}{833\cancel{0}} =$$
$$= \frac{5\cdot \cancel{476}\,4}{\cancel{119}} = 5\cdot 4 = 20$$

$$81^{\frac{3}{4}} \cdot 0{,}125^{-\frac{1}{3}} = 81^{\frac{3}{4}}\cdot\left(\frac{1}{8}\right)^{-\frac{1}{3}} = 81^{\frac{3}{4}}\cdot 8^{\frac{1}{3}} =$$
$$= \left(3^4\right)^{\frac{3}{4}}\cdot\left(2^3\right)^{\frac{1}{3}} = 3^{4\cdot\frac{3}{4}}\cdot 2^{3\cdot\frac{1}{3}} = 3^3\cdot 2^1 =$$
$$= 27\cdot 2 = 54$$

$$4^{\frac{1}{5}} \cdot 16^{\frac{9}{10}} = 4^{\frac{1}{5}}\cdot\left(4^2\right)^{\frac{9}{10}} = 4^{\frac{1}{5}}\cdot 4^{\frac{9}{5}} = 4^{\frac{1}{5}+\frac{9}{5}} = 4^{\frac{10}{5}} =$$
$$= 4^2 = 16$$

Свойства степеней

1. $a^1 = a$
2. $a^0 = 1$ при $a \neq 0$[a]
3. $a^{-1} = \dfrac{1}{a}$, $a^{-n} = \dfrac{1}{a^n}$
4. $a^m \cdot a^n = a^{m+n}$
5. $\dfrac{a^m}{a^n} = a^{m-n}$
6. $(a^m)^n = a^{mn}$
7. $a^n \cdot b^n = (a \cdot b)^n$
8. $\dfrac{a^n}{b^n} = \left(\dfrac{a}{b}\right)^n$

[a] Выражение 0^0 не определено.

$$a^n = \underbrace{a \cdot a \cdot \ldots \cdot a}_{n \text{ раз}}$$

$$n \in \mathbb{N}$$

$$\lim_{x \to 0} \frac{1}{x} = \infty$$

Степень отрицательного числа

Пусть $n \in \mathbb{Z}$ — чётное значение. Тогда

$$(-a)^n = a^n = b > 0$$

Если $n \in \mathbb{Z}$ — нечётное, то

$$(-a)^n = -a^n = b < 0$$

$$\underbrace{2{,}4}_{6 \cdot 0{,}4}{}^{\frac{3}{7}} \cdot \underbrace{0{,}16}_{=0{,}4^2}{}^{-\frac{5}{7}} \cdot 6^{\frac{4}{7}} = (6 \cdot 0{,}4)^{\frac{3}{7}} \cdot (0{,}4^2)^{-\frac{5}{7}} \cdot 6^{\frac{4}{7}} =$$

$$= 6^{\frac{3}{7}} \cdot 0{,}4^{\frac{3}{7}} \cdot 0{,}4^{-\frac{10}{7}} \cdot 6^{\frac{4}{7}} = 6^{\frac{3}{7} + \frac{4}{7}} \cdot 0{,}4^{\frac{3}{7} - \frac{10}{7}} =$$

$$= 6^1 \cdot 0{,}4^{-1} = 6 \cdot \frac{10}{4} = 6 \cdot \frac{5}{2} = 3 \cdot 5 = 15$$

$$\underbrace{52}_{13 \cdot 4}{}^{7{,}8} \cdot 13^{-5{,}8} : \underbrace{16}_{4^2}{}^{3{,}9} = (13 \cdot 4)^{7{,}8} \cdot 13^{-5{,}8} : (4^2)^{3{,}9} =$$

$$= 13^{7{,}8} \cdot 13^{-5{,}8} \cdot 4^{7{,}8} : 4^{7{,}8} = 13^2 \cdot 1 = 169$$

$$\frac{1089\cdot 70^3}{231000} = \frac{1089\cdot 7^3\cdot \cancel{1000}}{231\cancel{000}} = \frac{\cancel{11}\cdot 99\cdot 49\cdot \cancel{7}}{\cancel{11}\cdot \cancel{21}\ 3} =$$

$$70^3 = (7\cdot 10)^3 = 7^3\cdot 10^3$$

$$= 33\cdot 49 = 1617$$

$$(27\cdot 2^{-4})\cdot(6\cdot 5^{-3}) = 27\cdot \frac{1}{2^4}\cdot 6\cdot \frac{1}{5^3} =$$

$$= \frac{27\cdot \cancel{6}\ 3}{\underbrace{2^4}_{2^3\cdot\cancel{2}}\cdot 5^3} = \frac{27\cdot 3}{2^3\cdot 5^3} = \frac{81}{10^3} =$$

$$= \frac{81}{1000} = 0{,}081$$

$$(\underbrace{1{,}96}_{1{,}4^2}{}^{1{,}5}\cdot 1{,}4^{-1{,}6})^{\frac{5}{7}} = \left((1{,}4^2)^{1{,}5}\cdot 1{,}4^{-1{,}6}\right)^{\frac{5}{7}} =$$

$$= \left(1{,}4^3\cdot 1{,}4^{-1{,}6}\right)^{\frac{5}{7}} = \left(1{,}4^{1{,}4}\right)^{\frac{5}{7}} = 1{,}4^{\frac{14}{10}\cdot\frac{5}{7}} =$$

$$= 1{,}4$$

$$\frac{\left(30^{\frac{1}{12}}\cdot \overset{7^2}{49}{}^{\frac{1}{24}}\right)^{72}}{21^6} = \frac{\left(30^{\frac{1}{12}}\cdot(7^2)^{\frac{1}{24}}\right)^{72}}{21^6} =$$

$$= \frac{\left(30^{\frac{1}{12}}\cdot 7^{\frac{1}{12}}\right)^{72}}{21^6} = \frac{\left(210^{\frac{1}{12}}\right)^{72}}{21^6} = \frac{210^6}{21^6} =$$

$$= \left(\frac{210}{21}\right)^6 = 10^6 = 1000000$$

(ЕГЭ-2013). $\dfrac{a^{3{,}21}\cdot a^{7{,}36}}{a^{8{,}57}}$ при $a=12$

$$\frac{a^{3,21+7,36}}{a^{8,57}}=\frac{a^{10,57}}{a^{8,57}}=a^{10,57-8,57}=a^{2}$$

$$12^{2}=144$$

$$\frac{(4p^2)^3\cdot(7q)^4}{(28p^3q^2)^2}=\frac{4^3\cdot(p^2)^3\cdot 7^4\cdot q^4}{28^2\cdot(p^3)^2\cdot(q^2)^2}=$$

$$=\frac{4^3\cdot\cancel{p^6}\cdot 7^4\cdot\cancel{q^4}}{\underbrace{28^2}_{7\cdot 4}\cdot\cancel{p^6}\cdot\cancel{q^4}}=\frac{4^3\cdot 7^4}{7^2\cdot 4^2}=7^{4-2}\cdot 4^{3-2}=$$

$$=7^2\cdot 4^1=49\cdot 4=196$$

$\dfrac{5^{n+3}}{\underbrace{35}_{5\cdot 7}{}^{n+4}}$ при $7^{n+4}=\dfrac{1}{65}$

$$\frac{5^{n+3}}{5^{n+4}\cdot 7^{n+4}}=5^{n+3-(n+4)}\cdot\frac{1}{7^{n+4}}=$$

$$=5^{-1}\cdot\frac{1}{1/65}=\frac{1}{5^1}\cdot\frac{1\cdot 65}{1}=\frac{1}{5}\cdot 65=13$$

$$\frac{(13t)^3-13t^2}{t^2-169t^3}=\frac{13^3\cdot t^3-13\cdot t^2}{t^2-169\cdot t^3}=$$

$$= \frac{13 \cdot \cancel{t^2}(13^2 \cdot t - 1)}{\cancel{t^2}(1 - 169 \cdot t)} = \frac{13 \cdot (\cancel{169t - 1})}{-(\cancel{169t - 1})} =$$

$$= \boxed{-13}$$

$\frac{a}{b}$ при $\underbrace{\frac{2a+b}{7a-4b} = \frac{1}{2}}$

$$(2a+b) \cdot 2 = (7a-4b) \cdot 1$$

$$4a + 2b = 7a - 4b$$

$$4a - 7a = -4b - 2b$$

$$-3a = -6b \quad | : (-3)$$

$$a = 2b \quad | : b$$

$$\frac{a}{b} = \boxed{2}$$

или: $a = 2b$ тогда

$$\frac{a}{b} = \frac{2b}{b} = 2$$

$$\left((7x - 3y)^2 - 49x^2 - 9y^2\right) : 6xy =$$

$$= (\cancel{49x^2} - 2 \cdot 7x \cdot 3y + \cancel{9y^2} - \cancel{49x^2} - \cancel{9y^2}) : 6xy =$$

$$= \frac{-\cancel{2} \cdot 7 \cdot \cancel{3} \cdot \cancel{xy}}{\cancel{6}\,\cancel{xy}} = \boxed{-7}$$

Найдите значение выражения:

▶ 43 (ЕГЭ-2013). $\dfrac{a^{3,33}}{a^{2,11}\cdot a^{2,22}}$ при $a=\dfrac{2}{7}$ 3,5

▶ 44. $\dfrac{b^{0,4}\cdot b^{\frac{7}{5}}}{b^{-2\frac{1}{5}}}$ при $b=\sqrt{2}$ 4

▶ 45. $\dfrac{(7x^4)^3\cdot(2y)^2}{(14x^6y)^2}$ 7

▶ 46. $\dfrac{(105mn^6)^3}{(5n^9)^2\cdot(21m)^3}$ 5

▶ 47. $\dfrac{3^{m+5}}{12^{m+4}}$ при $2^{m+4}=\dfrac{1}{3}$ 27

▶ 48. $\dfrac{21^{n+1}}{7^{n-1}}$ при $3^{n+1}=\dfrac{5}{7}$ 35

▶ 49. $\dfrac{7u^2-(7u)^3}{49u^3-u^2}$ −7

▶ 50. $\dfrac{(25c^3)^2+25c^3}{c^3+25c^6}$ 25

▶ 51. $\dfrac{a+3b+16}{a-b+8}$ при $\dfrac{a}{b}=5$ 2

▶ 52. $\dfrac{b}{a}$ при $\dfrac{a+2b+24}{a+b+4}=6$ −1,25

▶ 53. $\left((11p+4q)^2-121p^2-16q^2\right):22pq$ 4

▶ 54. $3ab:\left(4a^2+9b^2-(2a-3b)^2\right)$ 0,25

$\boxed{44}$

$$\frac{b^{0,4}\cdot b^{\frac{7}{5}}}{b^{-2\frac{1}{5}}}=\frac{b^{\frac{2}{5}}\cdot b^{\frac{7}{5}}}{b^{-\frac{11}{5}}}=b^{\frac{2}{5}+\frac{7}{5}+\frac{11}{5}}=b^{\frac{20}{5}}=$$

$$=b^4=(\sqrt{2})^4=\left((\sqrt{2})^2\right)^2=2^2=4$$

$\boxed{52}$

$$\frac{a+2b+24}{a+b+4}=\frac{6}{1}$$

$$a+2b+24=6\cdot(a+b+4)$$

$$a+2b+24=6a+6b+24$$

$$a-6a=6b+\cancel{24}-2b-\cancel{24}$$

$$-5a=4b \quad |:4$$

$$-\frac{5}{4}a=b \quad |:a$$

$$-\frac{5}{4}=\frac{b}{a}$$

$\frac{b}{a}=?$

=> ответ: −1,25

$\boxed{53}$

$$\frac{(11p)^2+2\cdot11p\cdot4q+(4q)^2-16q^2-121p^2}{22pq}=$$

$$=\frac{\cancel{121p^2}+2\cdot11\cdot4\cdot pq+\cancel{16q^2}-\cancel{16q^2}-\cancel{121p^2}}{22pq}=$$

$$=\frac{\cancel{22}\cdot4\cdot\cancel{pq}}{\cancel{22}\cdot\cancel{pq}}=4$$

Найти значение $\underbrace{(4x-11)(4x+11)}-16x^2-4x+51$, если $x=-20$.

$$(4x)^2-11^2-16x^2-4x+51=\cancel{16x^2}-\underline{121}-\cancel{16x^2}-4x+\underline{51}=$$

$$=-4x-70=-4\cdot(-20)-70=80-70=10$$

Найти значение $\dfrac{13}{a^2b+ab^2}:\dfrac{52}{ab^3-a^3b}$, если $a=3{,}12$, $b=7{,}56$.

$$\frac{\cancel{13}}{\cancel{ab}(a+b)}\cdot\frac{\cancel{ab}(b^2-a^2)}{\cancel{52}\ 4}=\frac{(b-a)\cancel{(b+a)}}{\cancel{(a+b)}\cdot 4}=$$

$$=\frac{b-a}{4}=\frac{7{,}56-3{,}12}{4}=\frac{4{,}44}{4}=1{,}11$$

Найти значение $\dfrac{r^3+s^3}{4s}\cdot\dfrac{(r+s)^2-(r-s)^2}{r^3-r^2s+rs^2}$, если $r=\sqrt{11}+3$, $s=3-\sqrt{11}$.

$$\frac{(r+s)\cdot\cancel{(r^2-rs+s^2)}}{4s}\cdot\frac{\cancel{r^2}+2rs+\cancel{s^2}-\cancel{r^2}+2rs-\cancel{s^2}}{r\cdot\cancel{(r^2-rs+s^2)}}=$$

$$=\frac{r+s}{\cancel{4s}}\cdot\frac{\cancel{4}\cdot\cancel{r}\cdot\cancel{s}}{\cancel{r}}=r+s=$$

$$=\cancel{\sqrt{11}}+3+3-\cancel{\sqrt{11}}=6$$