The stages of testing statistical hypotheses

Example 1. The coin is examined for symmetry (heads and tails are equally likely). The coin is flipped 1000 times, the theoretical probability of the outcome is p = 1/2 the expected number of the coat of arms is 500. If the coat of arms fell out 511 times in the experiment, can we consider the coin symmetrical? (J. Pollard vol.1, pp.109, 139-...)

Typical Procedure	***For Example 1***
We need	Probability estimation: $p = p_0$
1. to select the significance level α	α=0,05
2. to describe the statistical model	
3. to formulate the main hypothesis H_0 and an alternative one H_1	H_0: $p=1/2$ H_1: $p \neq 1/2$ here is the case of a two-sided criterion
4. select a criterion statistic (criterion) whose behavior is known	The distribution of r. v. $T = (X - np)/\sqrt[2]{npq}$ approaches *N(0,1)* with increasing n;
5. To test the null hypothesis, a critical area is determined. If the value of the accepted statistical criterion falls within this area, then the null hypothesis is rejected. The probability of falling into this area, provided that H_0 true, equal toα	critical area: $\|T\| > 1{,}96$ see the function ML: `X = norminv(P,mu,sigma)` Note that in the case of the right-hand criterion, we choose: $X_{crit}=F^{-1}(P_{crit})$; $P_{crit}=0.95$; for two-side: $P^{l}_{crit}=0.025$; $P^{r}_{crit}=0.975$; In the case under consideration, the one-sided criterion is due to the absence of negative
6. calculate the value of a statistical criterion	$T=(511-500)/\sqrt[2]{250}=0{,}696$
7. draw conclusions. If the value of the criterion statistics in the critical area is rejected by the null hypothesis, otherwise it should be accepted H_0	The value of the criterion statistics does not fall into the critical area, there is no reason to consider the coin asymmetric.

Remark 1. *It is known that the probability that the number of drops of the coat of arms is located between r_1 and r_2, inclusive, equal to* $\sum_{r=r_1}^{r_2} \binom{n}{r} p^r q^{n-r}$ *and for n>20 the central limit theorem allows us to replace this sum with the expression:*

$$\int_{\alpha_1}^{\alpha_2} \frac{1}{\sqrt{2\pi}} exp(-x^2/2)dx,\ \alpha_1 = (r_1 - np - 1/2)/\sqrt{npq},\ \alpha_2 = (r_2 - np + 1/2)/\sqrt{npq}$$

Remark 2. *The obtained value of the criterion is not a proof of the* ***validity*** *of the null hypothesis, but evidence that there is no reason to reject* H_0.

Remark 3 *Stable criteria are those for which a moderate deviation of the data from the selected (assumed) statistical model (step 2) has little effect on the reliability of the conclusions. Thus, criteria based on the normality of population distributions are stable, but criteria for hypotheses about the values of variance are not.*

Remark 4*. Nonparametric criteria (free from distribution) are inferior in power (probability of rejection of the null hypothesis) to criteria based on normality, but they can be used in situations where distributions are not known in advance.*

Why does the assumed statistical model of a number of criteria assert the normality of the considered population?

Limiting relationships of probability distributions

I. Binomial distribution and Normal

In probability theory, it was proved that the binomial distribution can be approximated by a normal one. So instead of calculating the probability of a *coat of arms* (герб) falling out, enclosed between r_1 и r_2, equal $\sum_{r=r_1}^{r_2} \binom{n}{r} p^r q^{n-r}$ we use a normal law approximation with a probability density equal to $f(x) = \frac{1}{\sqrt{2\pi}} exp(-x^2/2)$ (for n>20), see the remark 1.

II. Binomial distribution and Poisson distribution

Definition. Binomial distribution and Poisson distribution

P(X=r)= $e^{-\lambda}\lambda^r/r!$ E(X)= λ, D(X) = λ.

The distribution according to Poisson's law can be obtained from the binomial, provided that p, q ≈1, close to one, so the binomial average $np=\lambda$. Additional confirmation of the validity of this approach follows from the binomial distribution

P(X=r)=$\binom{n}{r}$ $p^r q^{n-r}$, n – big enough; p – small; $np=\lambda$ – small, fixed. The limit value of the probability at $n \to \infty, p \to 0$ equal P(X=r)= $e^{-\lambda}\lambda^r/r!$, that is in the limiting case, it is Poisson.

III. The normal approximation of Poisson's law

It is known that the probability of r.v. X, enclosed in the interval $r_1 < X < r_2$ equal to

P($r_1 < X < r_2$)= $\sum_{r=r_1}^{r_2} e^{-\lambda}\lambda^r/r!$.

If $\lambda > 10$ the central limit theorem allows us to replace this sum with the expression

$\int_{\alpha_1}^{\alpha_2} \frac{1}{\sqrt{2\pi}} exp(-x^2/2)dx$, $\alpha_1 = (r_1 - \lambda - \mathbf{1/2})/\sqrt{npq}$, $\alpha_2 = (r_2 - \lambda + \mathbf{1/2})/\sqrt{\lambda}$

Note*, for the big* λ amendments $\lambda = \pm\mathbf{1/2}$ are neglected, similarly α_i is corrected in *remark 1*.

IV. Relationships distributions χ_ν^2 and Normal

If ν *is* big enough (ν >*30*), then the distribution of the value $Z = \sqrt{2Y} - \sqrt{2\nu - 1}$ is approximately normal with zero mean and unit variance. Distribution χ_ν^2 is used in estimating variance for sampling from a normal population and checking the dependence in the conjugacy [ˈkɒndʒʊgəsɪ] tables.

Example 2. In country Z, the height of adult men is described by a normal distribution with an average of 175.6cm and a standard deviation of 7.63cm. A man is randomly selected, what is the probability that his height will be in the range from 175cm to 185 cm.

Example 3. Check the following statement 1.

Statement 1. Let the r.v. be described by the distribution χ_1^2, that is, the square of the standard r.v. Show that for an arbitrary r.v. u, distributed normally, equality is valid for the one you set a:
P($\chi_1^2 > a^2$)= P($u < a$)+ P($u > a$)=2 P($u > a$)

Remark. You should use MatLab or any package (programming language)

Example 4. Construct the probability density functions of the standard normal distribution and the probability density of a Student with ten degrees of freedom. Identify X_c are the critical points for both density functions corresponding to the following expressions $P(x > X_c) = P(x < -X_c) = 0.025$. Specify these points on the graph.

Example 5. Find the upper and lower 2.5% points of the binomial distribution with parameters n=18, p= 0.4. Use the Poisson distribution to estimate commensurate critical regions.

Example 6. Find the upper and lower 5% points of the Poisson distribution λ=1.5 Use the Poisson distribution to estimate commensurate critical regions.

V. Polynomial distribution

Let there be independent tests of the k-dimensional bone (cube). The probability of getting n_1 – times the first face; n_2 – times the second; etc. and n_k – times k-th is equal to:

$$P(n_1, n_2, \cdots, n_k) = \frac{n}{n_1! n_2! \cdots n_k!} p_1^{n_1} p_1^{n_2} \cdots p_1^{n_k} \sum_{i=1}^{k} p_i = 1 \text{ и } \sum_{i=1}^{k} n_i = n.$$

probability p_i – of success of *i*–th face, (non-falling) failure: 1- p_i; mean = np_i; variance = $np_i(1 - p_i)$

Example 8. *The correct six-sided die is thrown 21 times. Find the probability that on the 21st throw: exactly one unit, exactly four fours, exactly two twos, exactly five fives, exactly six sixes and three threes.*

Practical Tasks: *Necessary to consider examples and answer the questions*