

Lab 6: Solving Nonlinear Systems and Equations. Integration and Differentiation. Applications.

All methods supported in MatLab related to the symbolic core can be identified with the command:

>>methods(sym)

If you find any typos, please write to me promptly via corporate email. nvkurbatova@sfedu.ru

Option 1:

Use the *fplot*, *fzero*, and *fsolve* functions.

Task 1.

Find all the intersection points of the two lines $y=\sin(\exp(x))$ and $g=0.6x^2-0.5$, Mark these points on the graph (closed shape marker, fill with color and choose the size at your discretion), and also write their coordinates in the title as follows: $A(x_1,y_1)$, $B(x_2,y_2)$; x_i,y_i – numeric values of the coordinates of the points, reduce the number of decimal places to two; place A and B in the axes near the intersection points of the curves using *gtext*.

Task 2.

Find all the points of intersection of two lines $y= x \cdot \sin(8x)$ and $g= x^5-x+0.5$ on the segment $[-1,1]$, having previously determined the abscissas of these points as the roots of the function $f= x \cdot \sin(8x) - (x^5-x+0.5)$. Mark the intersection points of the lines on the graph with a triangular marker and also label their coordinates with the *text* function.

Task 3.

Solve the system of equations.
$$\begin{cases} y = x \cdot \sin(x) \\ y = 1 - x^2 \end{cases}$$

Check the accuracy of your solution. Provide a graphical interpretation of the solution.

Comment. In Matlab, a polynomial can be represented as a symbolic expression, a string. However, some operations with polynomials are based on such a vector representation of the polynomial $P=[a(n), a(n-1), \dots, a(1), a(0)]$, for example, the function *polyval*(P,X) calculates the value of the polynomial at the point X, the function *roots* ([a(n), a(n-1), ..., a(1), a(0)]) numerically finds the roots of a polynomial, the function *poly*([x1, x2, ..., xk]) calculates the coefficients of the polynomial given the roots [x1, x2, ..., xk], thereby determining the type of the polynomial $g(x)=a(n)x^k + a(n-1)x^{k-1} + a(1)x + a(0)$ see Help

Use the *roots*, *poly*, and *polyval* functions.

Task 4.

Find all roots of the fifth-order polynomial $p(x)=x^5+x^2-10x-4.5$, Build a polynomial from the roots and check if it matches the original.

Sign the obtained real roots on the graph $p(x)$, highlighting them with the size and shape of the marker, and determine the accuracy with which they were found.

Task 5.

Find the type of a fifth-order polynomial if its roots are known: 1 - multiplicity 2; as well as simple roots: 3, 7, 9. Plot the graph of the function of the resulting polynomial, provide explanatory information.

Task 6.

Plot the graph of the function $y=\sin(x)$ for $x=-2\pi:0.1:2\pi$ using two methods, as well as the graphs of its first and second derivatives on the same axes, and provide a legend. The first method is numerical differentiation, the second is symbolic.

Task 7.

Find the local maxima of the function $y=\exp(\sin(x^3))$ on the interval $[-2,2]$. Determine the derivative $g(x)=y'(x)$ on the same interval, plot both functions in the same window, and provide a legend.

Task 8.

Determine numerically the area under a portion of the curve $y=\exp(-x)\cdot\sin(x)-(x-2)$, defined on the interval $[-2,2]$, which is located above the x-axis and intersects it. To calculate the integral, use the trapezoidal formula $\text{trapz}(x,y)$. Determine a partitioning step such that the calculation accuracy does not exceed $1.e-6$. Mark and label the limits of integration on the graph, and fill in the required area. Add a figure caption: (use LaTeX notation) $S = \int_a^b y(x) dx$

Task 9.

Calculate the definite integral of the function $y=x\cdot\sin(8x)-(x^5-x+0.5)$ with integration limits of -0.6, 0.6 numerically and using the *int* command of the symbolic kernel; achieve agreement of the results with an accuracy of $0.1e-4$.

Task 10.

Find the area enclosed by the lines $y=\exp(\sin(x))$ and $y=-x^2+8$. Fill the enclosed area with magenta, and mark the intersection points and their values on the graph.

Comment. In the symbolic core of Sym(bolic), the following functions work similarly to Maple: *subs*, *simplify*, *factor*, *collect*, *expand*, *taylor*, functions of summation and product of members of a series *symsum*, *symprod*.

Option 2:

Use the fplot, fzero, and fsolve functions.

Task 1.

Find all the intersection points of two lines $y=\cos(\exp(x))$; $g=-x^2+4$, Mark these points on the graph (choose the marker style and size yourself), and also label their coordinates using the text function, reducing the number of decimal places to two.

Task 2.

Find all the intersection points of two lines $y=\sin(2x)$; $g=-x^2+3$ on the segment $[-2, 2]$, first determine the abscissas of these points as the roots of the function $f = \sin(2x) + (x^2-3)$. Mark the intersection points of the lines on the graph with a closed marker, fill the marker with the color $[0.3 \ 0.2 \ 0.6]$, and also label their coordinates with the text function.

Task 3.

Solve the system of equations.
$$\begin{cases} y = \sin(\exp(x)) \\ y = -2 + x^2 \end{cases}$$

Check the accuracy of your solution. Provide a graphical interpretation of the solution.

Comment. In Matlab, a polynomial can be represented as a symbolic expression, a string. However, some operations with polynomials are based on such a vector representation of the polynomial $P=[a(n), a(n-1), \dots, a(1), a(0)]$, for example, the function `polyval(P,X)` calculates the value of the polynomial at the point X , the function `roots([a(n), a(n-1), \dots, a(1), a(0)])` numerically finds the roots of a polynomial, the function `poly([x1, x2, ..., xk])` calculates the coefficients of the polynomial given the roots $[x1, x2, \dots, xk]$, thereby determining the type of the polynomial $g(x)=a(n)x^k+a(n-1)x^{k-1}+a(1)x+a(0)$ see Help

Use the roots, poly, and polyval functions.

Task 4.

Find all roots of the fourth-order polynomial $p(x)=x^4+x^2-10x-1.5$. Build a polynomial from the roots and check if it matches the original.

Sign the obtained real roots on the graph $p(x)$, highlighting them with the size and shape of the marker, and determine the accuracy with which they were found.

Task 5.

Find the type of a fourth-order polynomial if its roots are known: 2 - multiplicities 2; as well as simple roots: 3 and 5. Plot the graph of the function of the resulting polynomial on the interval $[-7,7]$, provide explanatory information.

Task 6.

Construct the graph of the function $y=x \cdot \sin^2(x)$ for $x=-2\pi:0.1:2\pi$ in two ways, as well as the graphs of its first and second derivatives on the same axes, provide a legend. The first method is numerical differentiation, the second is symbolic.

Task 7.

Find the local maxima of the function $y=x \cdot \sin^2(x)$ for $x=-2\pi:0.1:2\pi$, Determine the derivative $g(x)=y''(x)$ on the same segment, plot both functions in one window, provide a legend.

Task 8.

Determine numerically the area under a portion of the curve $y=(1+x^2) \cdot \sin(x)+(x^2-3)$, defined on the interval $[0,5]$, which is located above the x-axis and intersects it. To calculate the integral, use the trapezoidal formula `trapz(x,y)`. Determine a partitioning step such that the calculation accuracy does not exceed $1.e-6$. Mark and label the limits of integration on the graph, and fill in the required area. Add a figure caption: (use LaTeX notation). $S = \int_a^b y(x) dx$

Task 9.

Calculate the definite integral of the function $y= x \sin(x)$ with integration limits -1 and 8 numerically and using the `int` command of the symbolic kernel; achieve agreement of the results to within $0.1e-4$.

Task 10.

Find the area enclosed between the curves $y= \sin(x) \cdot (1+x^2)$ and $g= x^2-3$ with intersection points on the segment $[-7, -3]$. Fill the closed area with cyan, mark the intersection points and their values on the graph.

Comment.In the symbolic core of Sym(bolic), the following functions work similarly to Maple: subs, simplify, factor, collect, expand, taylor, functions of summation and product of members of a series symsum, symprod.

Option 3:

Use the *fplot*, *fzero*, and *fsolve* functions.

Task 1.

Find all points of intersection of two lines $y=\sin(x) \cdot (1+x^2)$ and $g= x^2-3$ on the segment $[-2\pi, 4]$. Mark these points on the graph (choose the marker style and size yourself), and also sign their coordinates with the *text* function, reducing the number of decimal places to two.

Task 2.

Find all the intersection points of two lines $y=\sin(2 \cdot x)$; $g=-x^2+3$ on the interval $[-2\pi, 4]$, first determine the abscissas of these points as the roots of the function $f = \sin(2x) + (x^2-3)$. Mark the intersection points of the lines on the graph with a closed marker, fill the marker with the color $[0.2 \ 0.0 \ 0.8]$, and also label their coordinates with the address function *gtext*.

Task 3.

Solve the system of equations.
$$\begin{cases} y = -x \sin(2x) \\ y = 2 - x^2 \end{cases}$$

Check the accuracy of your solution. Provide a graphical interpretation of the solution.

Comment. In Matlab, a polynomial can be represented as a symbolic expression, a string. However, some operations with polynomials are based on such a vector representation of the polynomial $P=[a(n), a(n-1), \dots, a(1), a(0)]$, for example, the function *polyval*(P,X) calculates the value of the polynomial at the point X, the function *roots* ([a(n), a(n-1), ..., a(1), a(0)]) numerically finds the roots of a polynomial, the function *poly*([x1, x2, ..., xk]) calculates the coefficients of the polynomial given the roots [x1, x2, ..., xk], thereby determining the type of the polynomial $g(x)=a(n)x^k + a(n-1)x^{k-1} + a(1)x + a(0)$ see Help

Use the *roots*, *poly*, and *polyval* functions.

Task 4.

Find all roots of the 6th order polynomial $p(x)=x^6 + x^3 - x^2 - 3x - 1.5$. Build a polynomial from the roots and check if it matches the original.

Sign the obtained real roots on the graph $p(x)$, highlighting them with the size and shape of the marker.

Task 5.

Find the type of a 6th-order polynomial if its roots are known: 1 - multiplicity 2; as well as simple roots: 2, 3, 5, and 7. Plot the graph of the resulting polynomial function, select a representative segment, and provide explanatory information.

Task 6.

Construct the graph of the function $y = -x \sin(2x)$ in two ways. For $x = -2:0.1:2$, as well as the graphs of its first and second derivatives, provide a legend. The first method is numerical differentiation, the second is symbolic.

Task 7.

Find local minima of the function $y = -x \sin(2x)$ for $x = -2:0.1:2$. Determine the derivative $g(x) = y'(x)$ on the same segment, plot both functions in one window, and provide a legend.

Task 8.

Determine numerically the area under a portion of the curve $y = x \sin(2x)$, defined on the interval $[-2, 0.5]$, which is located above the x-axis and intersects it. To calculate the integral, use the trapezoid formula $\text{trapz}(x, y)$. Determine a partitioning step at which the calculation accuracy does not exceed $1.e-6$. Mark the integration limits on the graph, label them, and fill in the required area. Add a title: (use LaTeX notation). $S = \int_a^b y(x) dx$

Task 9.

Calculate the definite integral of the function $y = \sqrt{x^2 - 1}$ with integration limits 1 and 3 numerically and using the `int` command of the symbolic kernel; achieve agreement of the results to within $0.1e-4$.

Task 10.

Find the area enclosed between the lines $y = \sqrt{x^2 - 9}$ and $g = 2.5x - 7.5$ with intersection points on the segment $[3, 5]$. Fill the closed area with green, mark the intersection points and their values on the graph.

Comment. In the symbolic core of Sym(bolic), the following functions work similarly to Maple: subs, simplify, factor, collect, expand, taylor, functions of summation and product of members of a series symsum, symprod.

Option 4:

Use the fplot, fzero, and fsolve functions.

Task 1.

Find all the intersection points of two lines $y=6 \sin(x) / (1+x^2)$ and $g=x^2$ on the interval $[-2,2]$, mark these points on the graph (choose the marker style and size yourself), and also sign their coordinates using the gtext function, reducing the number of decimal places to two.

Task 2.

Find all the intersection points of two lines $y= \sin(\exp(x))$; $g=-x^2+3$ on the interval $[-3,2]$, first determine the abscissas of these points as the roots of the function $f= \sin(\exp(x)) + (x^2 - 3)$. Mark the intersection points of the lines on the graph with a closed marker, fill the marker with the color $[0.2 \ 0.8 \ 0]$, and also label their coordinates with the text function.

Task 3.

Solve the system of equations.
$$\begin{cases} y = 6 \sin(x)/(1+x^2) \\ y = 0.6 x^2 \end{cases}$$

Check the accuracy of your solution. Provide a graphical interpretation of the solution.

Comment. In Matlab, a polynomial can be represented as a symbolic expression, a string. However, some operations with polynomials are based on such a vector representation of the polynomial $P=[a(n), a(n-1), \dots, a(1), a(0)]$, for example, the function polyval(P,X) calculates the value of the polynomial at the point X, the function roots([a(n), a(n-1), ..., a(1), a(0)]) numerically finds the roots of a polynomial, the function poly([x1, x2, ..., xk]) calculates the coefficients of the polynomial given the roots [x1, x2, ..., xk], thereby determining the type of the polynomial $g(x)=a(n)x^k + a(n-1)x^{k-1} + a(1)x + a(0)$ see Help

Use the roots, poly, and polyval functions.

Task 4.

Find all roots of the 6th order polynomial $p(x)= 2x^6 -x^5 + 2x^4 + x^3 -x^2 -3x -12$. Build a polynomial from the roots and check if it matches the original.

Sign the obtained real roots on the graph $p(x)$, highlighting them with the size and shape of the marker, and determine the accuracy with which they were found.

Task 5.

Find the type of a 6th-order polynomial if its roots are known: 1 - multiplicity 4; as well as simple roots: 3 and 5. Plot the graph of the resulting polynomial function, select a representative segment, and provide explanatory information.

Task 6.

Construct the graph of the function $y=(x+3) \cdot \sin(2x)$ for $x=-2:0.1:1$, in two ways, as well as the graphs of its first and second derivatives on the same axes, provide a legend. The first method is numerical differentiation, the second is symbolic.

Task 7.

Find the local minimum of the function $y=(x+3) \cdot \sin(2x)$ for $x=-2:0.1:1$. Determine the derivative $g(x)=y'(x)$ on the same segment, plot both functions in one window, and provide a legend.

Task 8.

Determine numerically the area under a portion of the curve $y = -x \cdot \sin(2x)$, defined on the interval $[-2, 0.5]$, which is located above the x-axis and intersects it. To calculate the integral, use the trapezoid formula $\text{trapz}(x,y)$. Determine a partitioning step at which the calculation accuracy does not exceed $1.e-6$. Mark and label the integration limits on the graph, and fill in the required area. Add a figure caption: (use LaTeX notation). $S = \int_a^b y(x) dx$

Task 9.

Calculate the definite integral of the function $y = \sin(x) x^2$ with integration limits 0 and 1.5 numerically and using the int command of the symbolic kernel; achieve agreement of the results to within $0.1e-4$.

Task 10.

Find the area enclosed between the lines $y = -x^2$ and $g = x-2$ with intersection points on the segment $[-2, 2.5]$. Fill the closed area with the color $[0.5 \ 0 \ 0.5]$, mark the intersection points and their values on the graph.

Comment. In the symbolic core of Sym(bolic), the following functions work similarly to Maple: subs, simplify, factor, collect, expand, taylor, functions of summation and product of members of a series symsum, symprod.