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Scientific Computer Software

(MatLab)

Topic 9. Elements of differential and integral calculus

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Contents:

1) **Interpolation**
2) **Differentiation**
   a) Analytical differentiation
   b) Numerical differentiation
   c) Taylor and Fourier series
   d) Solving applied problems (i.e. pendulum)
3) **Integration:**
   a) Analytical integration Numerical integration.
   b) Trapezoid, Newton's methods, etc.
   c) Calculating the areas under the curve, inside the intersection of curves
4) **Special operators**

Почему целесообразны подгруппы?

Принцип МЕТРОНОМА в социуме. Мирзакарим Норбеков

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Derivative and numeric differentiation in ML

Let function $y=f(x)$ is calculated in the points $\{x_i\}$: $x_{i+1} = x_i+h$ (h – constant) chosen on interval [a, b] and a= x_1, b= x_n.

Numeric increment of a function on each subinterval $[x_i, x_{i+1}]$ is equal to $f(x_{i+1})- f(x_i)$.

Derivative of function:

continuous case:

$$f' = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h},$$

numeric case:

$$\boldsymbol{y} = \left\{\frac{f(x_i+h)-f(x_i)}{h}\right\}_{i=1}^{n}, \; h \text{ -small}$$

ML code: functions increment you can calculate using function **diff**

```
X=a:h:b;  F=f(X);
Y=diff(F)   % increment calculation
G=diff(Y) /h  % numeric analogue of the first order derivative f'
R=diff(G) /h  % analogue of the second order derivative f''
```

What about length of vectors Y,G,R? Continuous numeric

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Case of Y - matrix

[m,n]=size(Y) → [m-1,n]=size(diff(Y))

$f(x)$, $g(x)$, $p(x)$

$X=[x_1; x_2; \ldots; x_n]$ - column

$Y=[f(x_i), g(x_i), p(x_i)]$;

The first order derivatives for three functions:

diff(Y) / h

High order derivative. Size control!

diff(Y,n) – high order function increment

dim<n!

result= diff(Y,n), isempty(result)

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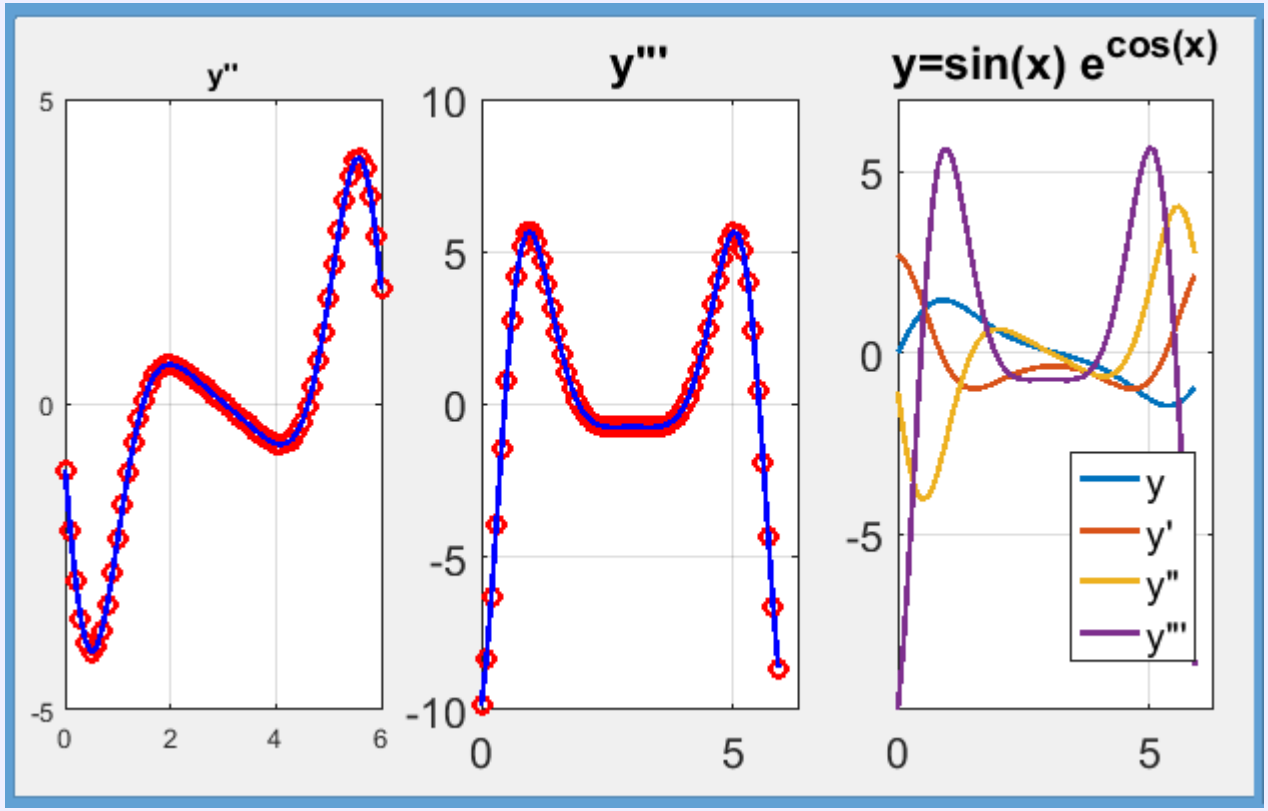
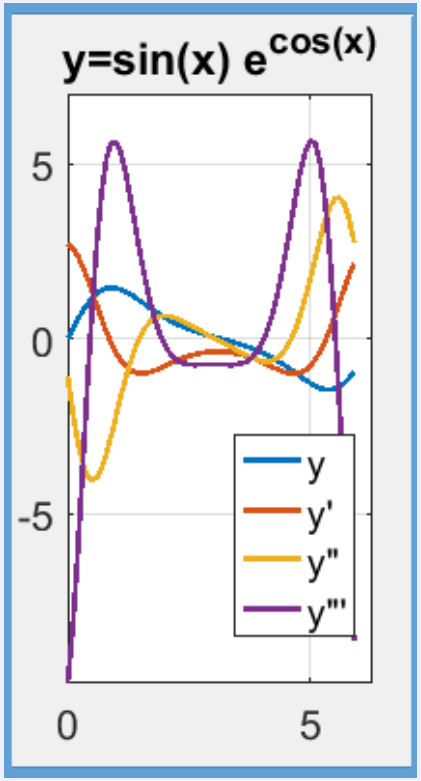
Numerical differentiation in pictures



- Explain how the changes in the design of the contents of the windows, as well as the axes, were obtained?
- How to build all the graphs at the same time, window 3?
- What is the relative size of the vectors y, y',y'',y''' and which one should be chosen when simultaneously constructing y, y',y'',y''' ?
- Let the vector x be of length n.

An example of numerical differentiation

```
1)  x=0:0.1:2*pi; n=length(x); h=0.1;
2)  f=sin(x).*exp(cos(x))
3)  fprime=diff(f,1)/h; fprime2=diff(f,2)/h^2;
4)   fprime3=diff(f,3)/h^3;
5)  subplot(1,3,3)
6)  xnew=x(end-3:-1:1)'  % row or column?
7)   fNew=f(end-3:-1:1)' % explain index!
8)    fprimeNew=fprime(end-2:-1:1)'
9)     fprime2New=fprime2(end-1:-1:1)'  % Why so?
10)     fprime3New=fprime3(end:-1:1)'
11)  YY=[fNew,fprimeNew,fprime2New,fprime3New]
12)  pl=plot(xnew,YY)
13)  set(pl,'linewidth',2)
14)  set(gca,'fontsize',16,'xlim',[0, 2*pi],...
     'ylim',[min(min(YY)), 7])
15)  leg=legend('y','y'''','y''''','y''''''')
16)  set(leg,'fontsize',14,'location','best')
17)  title('y=sin(x) e^{cos(x)}'), grid on
```



Explain: line 12 и plot(YY)!

Analytical differentiation

- Analytical differentiation is performed in the **SYMBOL** class
- All variables are declared, i.e.

syms var1 var2 var3;

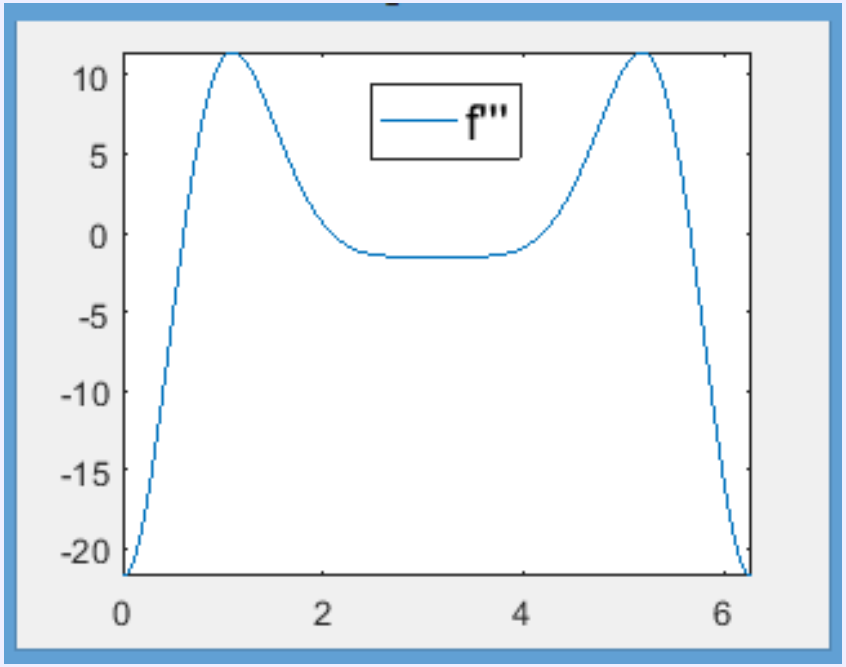
- When declaring variables, they are separated by spaces (not commas)!
- The function is constructed according to the rules of **MatLab** (external, subfunction, anonymous)
- The function can be specified in the syntax **Maple:**

Namefunction(args)= expression(args)

```
syms x y, z; % What is equal to  r?
f=cos(x)*y^2*exp(z), r=diff(f,2,z)
```

Class SYMBOL. An example of differentiation

```
clear;   syms x y
I. f is created according  Maple rules:
%f(x,y)=sin(x)*y*exp(cos(x))
g=diff(f,x,3)
fn=subs(g,y,2) %   ~  subs in Maple
figure
% the range of the argument is specified:
fplot(fn, [0,2*pi] )
lg=legend('f'''''''); set(lg,'fontsize',14)
II. f is set as a subfunction :
function g=f(x,y)
syms x y
g=sin(x)*y*exp(cos(x))
end
```



Syntax:

NewFunc=diff(Func,Args,order)

Func – symbol fuction

Args – variable of differentiation

order – order of differentiation

Taylor series

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$$f(x)=f(x_0)+\frac{f'(x_0)}{1!}(x-x_0)+\frac{f''(x_0)}{2!}(x-x_0)^2+\ldots$$
$$\ldots+\frac{f^{(n)}(x_0)}{n!}(x-x_0)^n+R_n(x)$$
$$R_n(x)=\frac{f^{(n+1)}(\xi)}{(n+1)!}(x-x_0)^{n+1},\quad \xi\in(x_0,x)$$

- In 1715, it was again received **by Brooke Taylor** (already!!! it was used in the XIV century in India, as well as in the XVII century, including by Newton).
- In the period (1698-1746), **Colin Maclaurin**, a Scottish mathematician, applied and popularized the case $x_0=0$
- **Joseph Louis Lagrange** — French mathematician (1736-1813) proposed the representation of the residual term.
- MAtLab use **Maclaurin** decomposition **default** $x_0=0$ and **Taylor** — $x_0=a$

Why do we need function decomposition as a series?

Syntax taylor decomposition in MatLab

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taylor(fun,var,Name,Value)

fun = symbolic function | symbolic expression

var= (default) if you do not specify a variable, the system will find the first one by itself symvar(fun,1) and it will spread it out at a point, otherwise we set it explicitly

'ExpansionPoint' x_0= 0 (default) | number | symbolic number | symbolic variable | symbolic function | symbolic expression

1) 'Order' = 6 (default) | positive integer | symbolic positive integer
2) 'OrderMode' = 'absolute' (default) | 'relative'
   for polynomial approximation

Value according to the need and the value options described in 1)-3)

ML functions of Symbolic class you can find using

>>methods(sym)

Function taylor in MatLab

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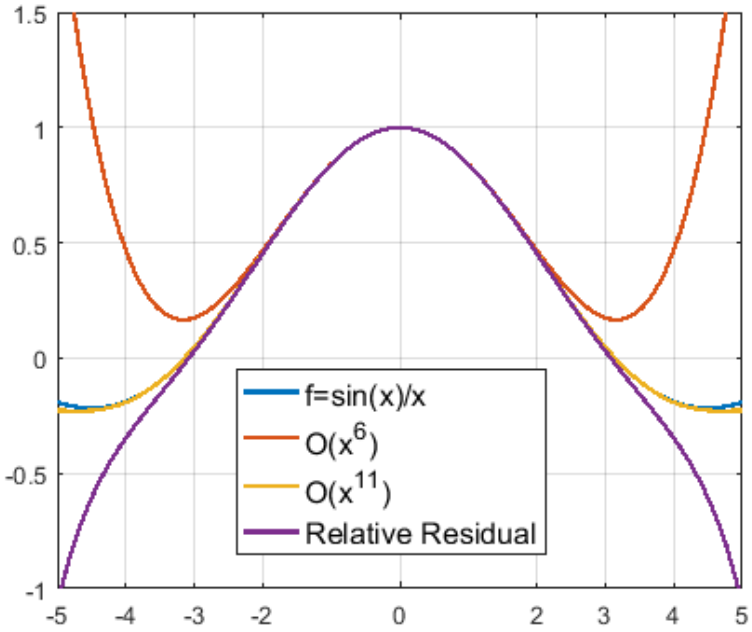
An example of a strategy for choosing **Value**

What exactly is the strategy?
We analyze the code and graphs!

```
1) clear; syms x
2) f = sin(x)/x
3) t6 = taylor(f, x) , % t9=... ???
4) tr=taylor(f, x, 'OrderMode', 'relative','order',7)
5) group=fplot([f t6 t9 tr ],'Linewidth',1.7)
6) xlim([-5 5]), ylim([-1 1.5]),
7) set(gca,'xtick',[-5 -4 -3 -2 0 2 3 4 5])
8) lg=legend('f=sin(x)/x','O(x^6)',...
   'O(x^{11})','RelativeSpec')
9) set(lg,'fontsize',12), grid on
```

Can we use it for integration during pre-decomposition?



A useful resource in the future ML

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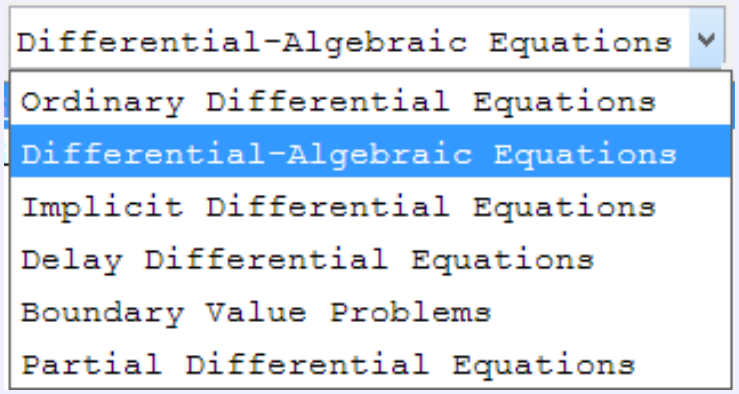
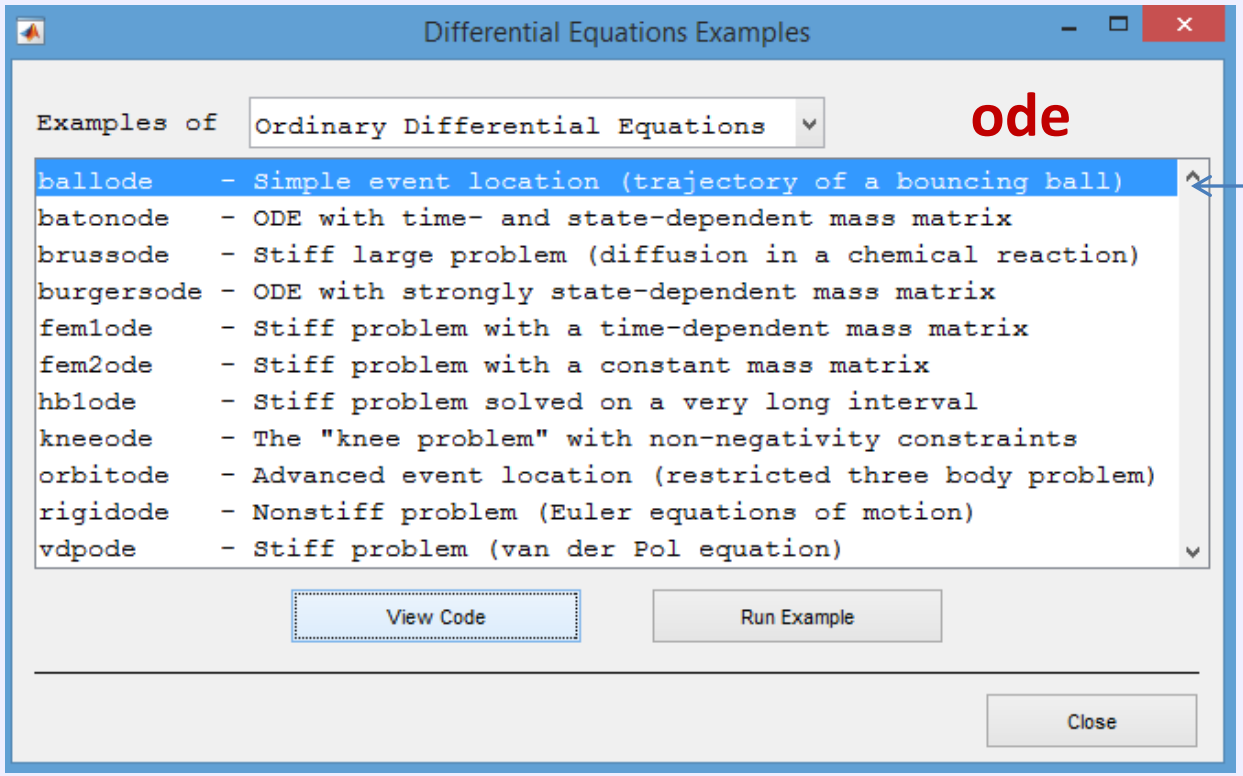
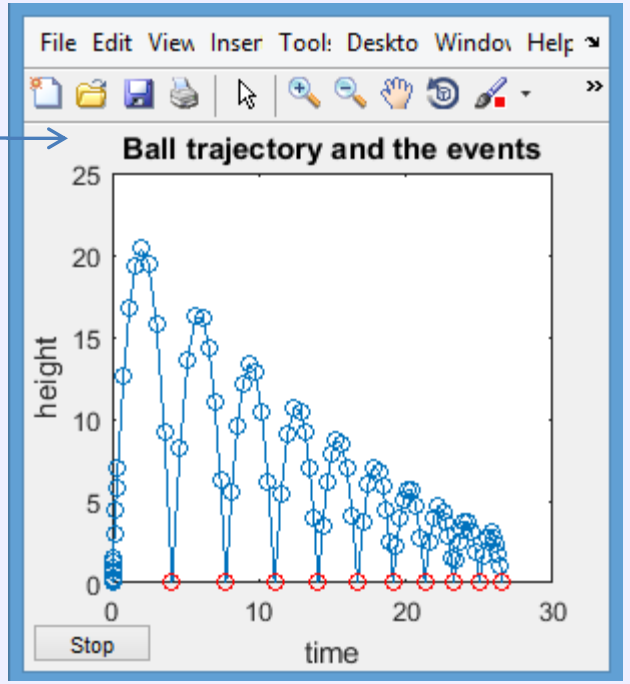
>>odeexamples(SECTION)

SECTION = 'ode'|'dae'|'ide'|'dde'|'bvp'|'pde'







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Assignment of sections of thematic groups in the resource **odeexamples**

- 'dae' – differential algebraic equations (DAE)
- 'ide' – implicit methods for solving the DAE problem
- 'dde' – differential equations with time delay
- 'bvp' – DAE under specified boundary conditions
- 'pde' – DAE in a partial differential equation

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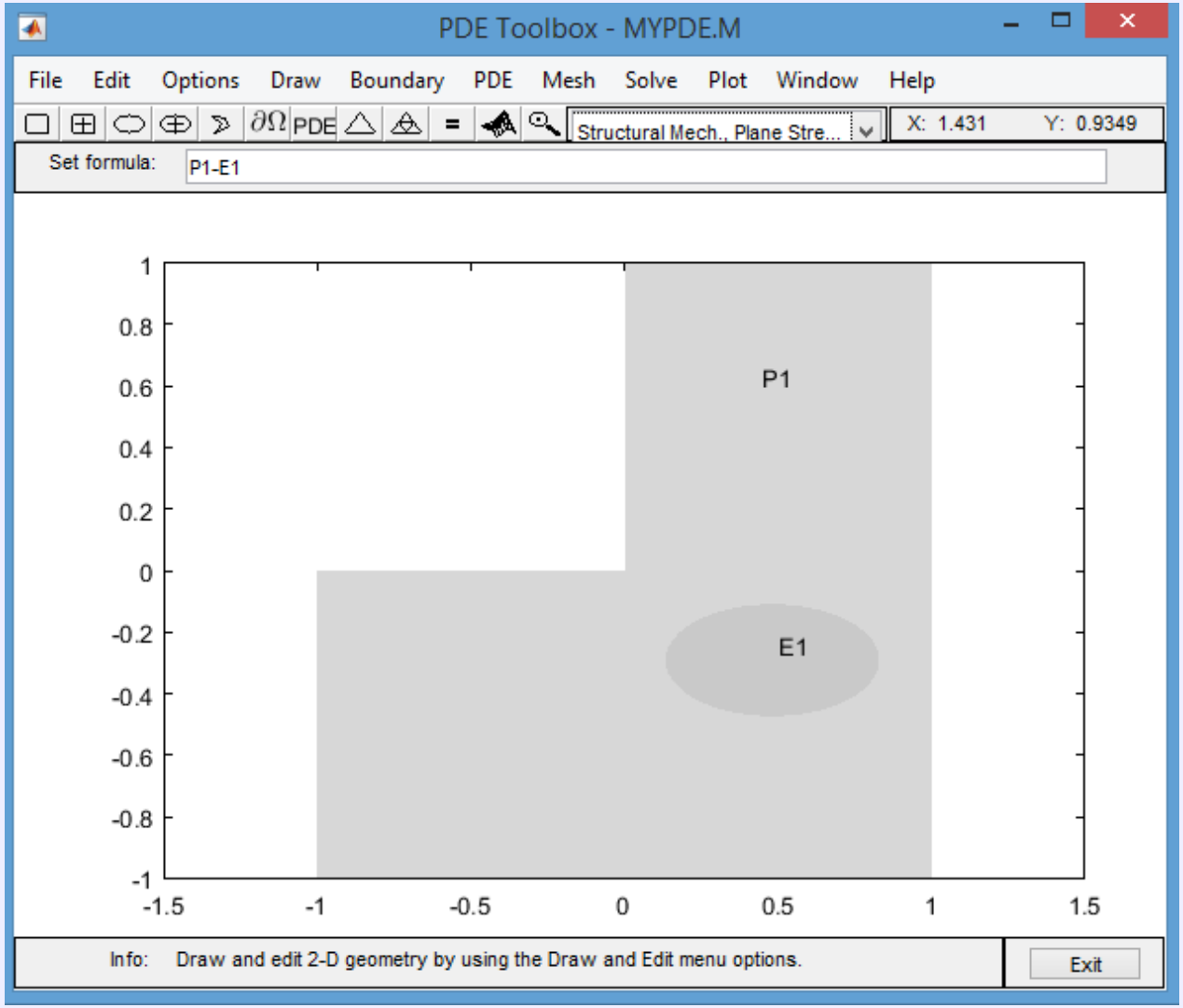
>> pdetool

PDE of 2order:

- Choosing the scope of application
- Constructing the PDE by setting the parameters
- Creating a geometry
- Boundary conditions
- Triangulate
- Improving triangulation as needed
- Solving
- Visualize the result



Class SYMBOL. Integration

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Function given as subfunction

```
arg=symvar(func); res=int(func)
res2=int(int(func,arg(1)),arg(3))
resDef=
 …int(int(func,arg(1),0,pi),arg(3),1,2)
```

function f=func

```
syms x y z
f=exp(y)*sin(x)/z
end
```

Func – according to Maple rules

```
f=exp(y)*sin(x)/z
arg=symvar(f)
res=int(f)
res2=int(int(f,arg(1)),arg(3))
resDef=int(int(f,arg(1),0,pi),arg(3),1,2)
```

Syntax:

Res=int(Func,Args)

Res=int(Func,Args,a,b)

Args=symvar(f)

Func – symbol function

Args – integration variable

a,b – limits

Note:

If no integration variable is specified, then integration is only based on the first variable

Res=int(Func)

Численное интегрирование в MatLab

Syntax:

Z = trapz(X,Y,DIM) -

the trapezoid method

X is the value of the arguments (vector)

Y is a matrix or vector of values

DIM indicator (1 ∨ 2) what does it mean?

Q = quad(FUN,a,b,tol,trace) –

Simpson method

FUN – *external* function or *anonymous*

we build according to the rules of vector operations!

a,b – **explain?**

trace>0 – output of intermediate values

The accuracy (tol) of the Simpson method in ML default: 1.e-6

Universal Function:

Res=integral(Func,Args)

Res=int(Func,Args,a,b) % may be old

Res=integral2(Func,Args,lims)

Res=integral3(Func,Args,lims)

Args=symvar(Func)

Func – symbol function

Args – no integration variable

a,b ∨ lims – limits of integration

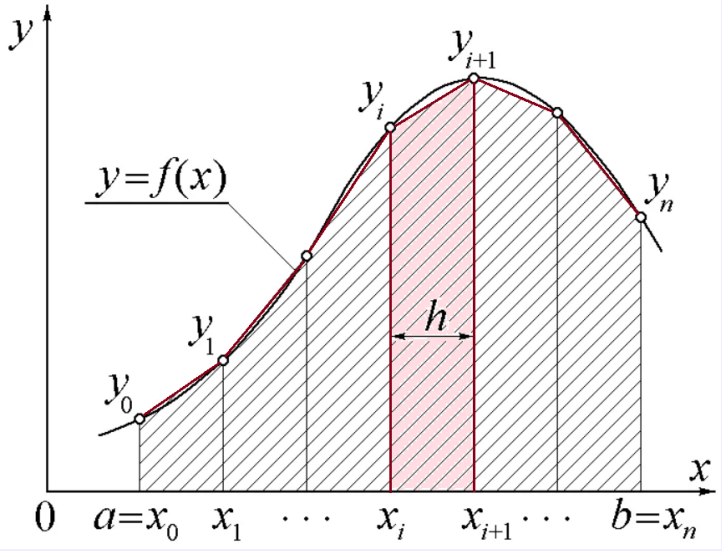
Note

If the integration variable is not specified, then integration is based on the first (in the sense of the ASCII code) of the variable

Res=integral(Func)

Trapezoid and Simpson formulas

Trapezoid:



$$\int_a^b f(x)\,dx \approx \frac{b-a}{2N}\sum_{n=1}^{N}\left(f(x_n)+f(x_{n+1})\right)$$

$$= \frac{b-a}{2N}\left[f(x_1)+2f(x_2)+\ldots+2f(x_N)+f(x_{N+1})\right]$$

$$h=\frac{b-a}{N}$$

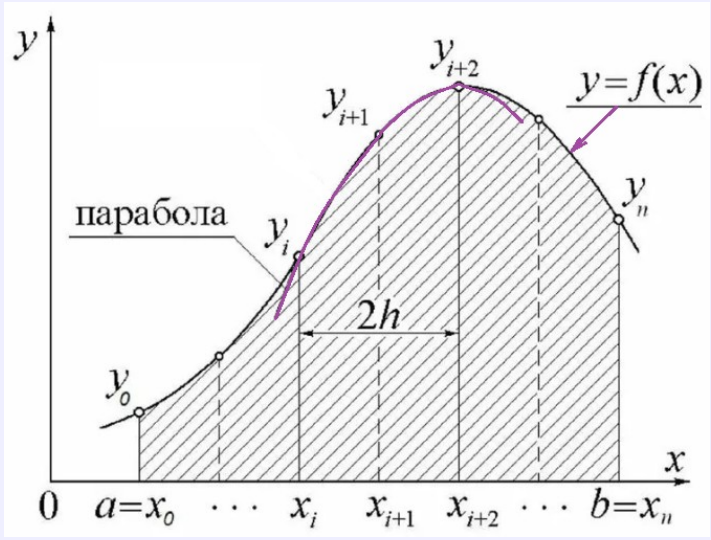
Simpson

$$I \approx \frac{b-a}{6}\left(f(a)+4f\left(\frac{a+b}{2}\right)+f(b)\right).$$

если $2N$ равных частей на $[a,b]$:

$$I \approx \frac{b-a}{6N}\left(f_0+4\left(f_1+f_3+\ldots+f_{2N-1}\right)+\ldots\right.$$
$$\left.2\left(f_2+f_4+\ldots+f_{2N-2}\right)+f_{2N}\right),$$

$$f_i = f\left(a+\frac{(b-a)i}{2N}\right).$$



Application. Calculate the area enclosed between the curves

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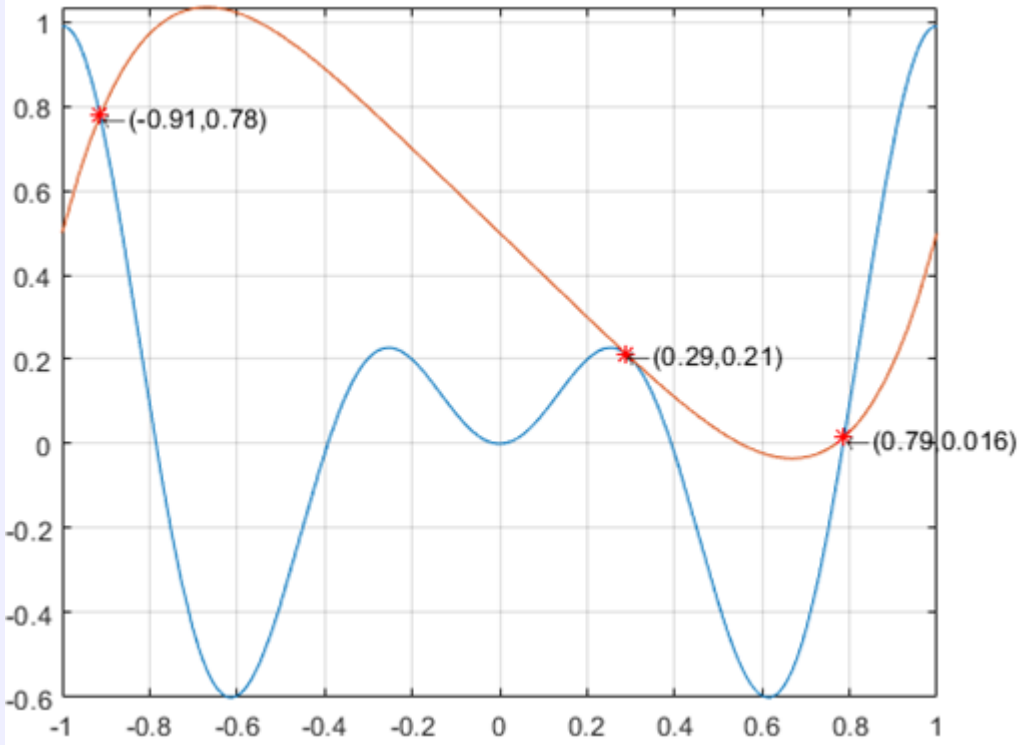

Think about the solution algorithm!

pause(10)

Have you thought about it?

Is the point (0.29,0.21) a touch point?

Why is this important?

Where do we start solving the problem?

Пример. **Example**

```
Clear, figure
%% строим графики
fplot('x*sin(8*x)',[-1 1]),hold on,
fplot('(x^5-x+0.5)',[-1 1]),grid on;
%% решаем, добавляем точки пересечения
x0=[-0.8,0.65;-0.2,0; 0.6,0]% нулевые
приближения
        for i=1:length(x0')
                [x{i},Fx]=fsolve(@F,x0(i,:))
                plot(x{i}(1),x{i}(2),'r*')
                xy=['(',num2str(x{i}(1),2),',',...
                        ,num2str(x{i}(2),2),')']
    text(x{i}(1),x{i}(2),['\leftarrow' xy])
      end % for
function f=F(x)   %% подфункция в этом же файле
f(1)=x(2)-x(1)*sin(8*x(1))
f(2)=x(2)-(x(1)^5-x(1)+0.5)
end
```

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Спасибо за внимание!

"What the vessel is filled with, it pours out of it!"

Russian folk proverb

And why suddenly **Russian** proverbs?